DOCUMENT RESUME

ED 190 751 CE 025 470

TITLE	Unified Technical Concepts. Module 3: Rate.
INSTITUTION	Technical Education Research Center, Waco, Tex.
SPONS AGENCY	Bureau of Occupational and Adult Education (DHEW/OE), Washington, D.C.
BUREAU NO	498AH70194
PUB DATE	79
GRANT	G007701952
NOTE	176p.: For related documents see CE 025 465-480.
AVAILABLE FROM	UTC Coordinator, Technical Education Research Center--Southwest, 4201 Lake Shore Dr., Waco, TX 76701 (Concept Modules, $0.80 each; Application and Supplementary Modules, $0.40 each; Notebook for Modules, $2.25; Instructor's Guide, $8.00)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Energy; Force; Kinetics; Learning Activities; Learning Modules; Mechanics (Physics); *Motion; *Physics; Power Technology; *Technical Education; *Time; Two Year Colleges; Units of Study

ABSTRACT

This concept module on rate is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module presents the concept of rate as it applies to mechanical, fluidal, electrical, and thermal systems. Included with the concept module are eleven application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Graphical Approach to Linear Motion, Velocity at Constant Acceleration, Displacement at Constant Acceleration, Acceleration of a Mass by Constant Force, Rotational Motion, Radial Acceleration and Centripetal Force, Flow Rate Measurement with a Venturi Meter, Electric Current, Frequency, Heating and Cooling Rate, and Rate of Heat Conduction. (LRA)

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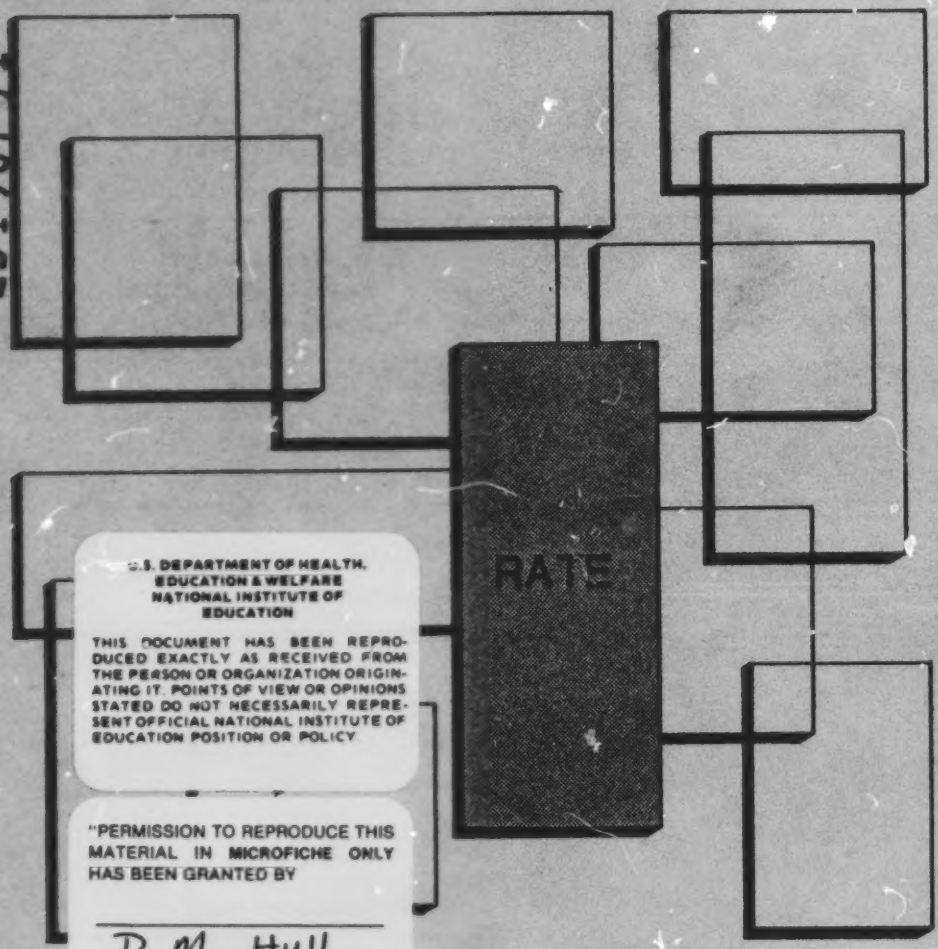
UTC

UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 3-0

ED190751



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INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems <u>force</u> and <u>torque</u> cause motion of mass.
- In fluidal systems <u>pressure difference</u> causes motion of fluids.
- In electrical systems <u>potential difference</u> causes motion of mass.
- In thermal systems <u>temperature difference</u> causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS

3-0	Rate
3M1	Graphic Approach to Linear Motion
3M2	Velocity at Constant Acceleration
3M3	Displacement at Constant Acceleration
3M4	Acceleration of a Mass by Constant Force
3M5	Rotational Motion
3M6	Radial Acceleration and Centripetal Force
3F1	Flow Rate Measurement with a Venturi Meter
3E1	Electric Current
3E2	Frequency
3T1	Heating and Cooling Rate
3T2	Rate of Heat Conduction

INTRODUCTION

If a car travels 120 miles in three hours, it travels an average of 40 miles every hour; the car's average speed is 40 miles per hour. Forty miles per hour is referred to as rate of motion or speed of the car. Both distance traveled and time required are used to determine rate of motion. Rate is a quantity that describes how rapidly an occurrance takes place.

This module presents the concept of rate as it applies to mechanical, fluidal, electrical, and thermal systems. In every case, rate expresses the ratio of a displacementlike quantity to an elapsed time:

- In a linear mechanical system, velocity is the ratio of distance traveled to elapsed time.
- In a rotational mechanical system, angular velocity is the ratio of angle of rotation to elapsed time.
- In a fluidal system, volume flow rate is the ratio of volume of fluid moved to elapsed time.
- In an electrical system, current is the ratio of charged transferred to elapsed time.
- In a thermal system, heat flow rate is the ratio of thermal energy transferred to elapsed time.

This module discusses these rates and others and presents the mathematical formulation and units used to express rates in all energy systems.

PREREQUISITES

The student should have completed Concept Module 2-0, "Work."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following rates and, and where applicable, express

their basic units both in SI and English systems of units:

Speed and velocity.
Acceleration.
Angular velocity.
Angular acceleration.
Volume flow rate.
Mass flow rate.
Electric current.
Heat flow rate.

- In a linear mechanical system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Displacement, elapsed time, velocity.
  Initial velocity, final velocity, elapsed time, acceleration.
  Mass, force, acceleration.

- In a rotational mechanical system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Angular displacement, elapsed time, angular velocity.
  Initial angular velocity, final angular velocity, elapsed time, angular acceleration.

- In a fluidal system, given all the quantities except one in each of the following groups, determine the unknown quantity:

  Volume of fluid moved, elapsed time, volume flow rate.
  Mass of fluid moved, elapsed time, mass flow rate.

- Given two of the following quantities in an electrical system, determine the third:

  Charge transferred.
  Elapsed time.
  Current.

- Given two of the following quantities in a thermal system, determine the third:

  Heat energy transferred.
  Elapsed time.
  Heat flow rate.

- State the general equation for rate, and explain how it applies to each of the following energy systems:

  Mechanical translational.
  Mechanical rotational.

Fluidal.
Electrical.
Thermal.

Concept Module 1-0, "Force," described forcelike quantities as the physical quantities that cause motion in all energy systems. Concept Module 2-0, "Work," described work done in an energy system in terms of changes that occur in that system as the result of the action of forcelike quantities present. A useful description of the affects of forcelike quantities in a system and work done in that system requires that the time necessary for the changes to occur be considered.

Movement is an occurrence common to every energy system. The rate of movement in each system can be determined by dividing the displacementlike quantity by the time in which the displacement takes place. The unifying formula used for calculation rate is shown in Equation 1.

$$\text{RATE} = \frac{\text{DISPLACEMENTLIKE QUANTITY}}{\text{TIME ELAPSED}} \qquad (1)$$

This module discusses the application of this general rate equation in mechanical, fluidal, electrical, and thermal systems.

MECHANICAL SYSTEMS

MECHANICAL TRANSLATIONAL RATES

The most fundamental rate in a mechanical system is speed. Speed is the rate at which an object moves without regard for the direction of motion of the object. Speed can be calculated by dividing the total length of the path of an object by the time required for the movement along that path. Speed indicates only the rate at which the object moves through some distance; it contains no information concerning the direction of the motion.

Velocity, rather than speed, is used whenever the direction of the moving body is important. Velocity specifies both the speed and direction of the motion. For example, "40 miles per hour" denotes a speed, while "40 miles per hour northeast" denotes a velocity.

When an object moves in a straight line, it is said to be in linear or translational motion. The object may stop or reverse direction, but its motion always lies along the same straight line. This discussion of mechanical translational rates will deal only with cases of straight-line motion. The term "speed" will be used when the direction of motion is not important; velocity will be used when both speed and direction are important. The formula for calculating speed (the magnitude of velocity) in a linear mechanical system is given by Equation 2.

$$v = \frac{d}{t} \qquad (2)$$

where

v = speed (magnitude of velocity).
d = displacement.
t = elapsed time.

The symbol v commonly is used for both speed and velocity. If the quantity identified as v includes a direction, it is the vector quantity velocity. If it does not include a direction, it is the scalar quantity speed.

Example A illustrates how Equation 2 is used to solve a problem.

EXAMPLE A: SPEED AND VELOCITY OF AN AUTOMOBILE

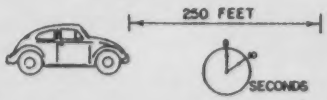
Given: An automobile travels 250 feet east in 10 seconds.

Find: Its speed and velocity.

Solution: Speed: $v = \frac{d}{t}$

$$= \frac{250 \text{ ft}}{10 \text{ s}}$$

$$v = 25 \frac{\text{ft}}{\text{s}}$$



The speed of the automobile is 25 ft/s. If direction is specified, we have a velocity.

Velocity = 25 ft/s, east.

If direction of the moving car in Example A were given as west, the car's velocity would be 25 ft/s, west.

Distance (displacement) often is measured in units such as feet, miles, or meters and time in units such as seconds, minutes, or hours. Common units for speed are summarized in Table 1.

TABLE 1: COMMON UNITS FOR SPEED	
System of Measure	Common Units for Speed
English	ft/s or miles/hour (44 ft/s = 30 mi/h)
SI	m/s or kilometers/hour (0.278 m/s = 1 km/h)

Example B illustrates the use of Equation 2 to determine elapsed time if speed and distance are known.

EXAMPLE B: TIME TO RUN A MILE

Given: A runner can average 13.4 mi/h.

Find: Time required to run one mile.

Solution: Speed $v = 13.34$ mi/h

$$= (13.34 \text{ mi/h})\left(\frac{44 \text{ ft/s}}{30 \text{ mi/h}}\right)$$

$$v = 19.56 \text{ ft/s}$$

From Equation 2: $t = \frac{d}{v}$

$$= \frac{5280 \text{ ft}}{19.56 \text{ ft/s}}$$

$$= 270 \text{ s}$$

$$= (270 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$t = 4.5 \text{ min}$$

Both Examples A and B involved average speed of an object. Speed was treated as if it were constant through the entire measured time. However, in many cases the speed of a moving object fluctuates as it moves in a fixed direction. The rate of change of linear speed is called "linear acceleration." A constant linear acceleration is calculated by using the relationship shown in Equation 3.

$$a = \frac{v_2 - v_1}{t} \qquad (3)$$

where

a = linear acceleration.
v_1 = initial speed (speed before change).
v_2 = final speed (speed after change).
t = time in which change takes place.

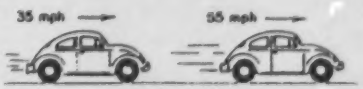
The units of acceleration are velocity units (m/s, ft/s, mi/h) divided by time units. The most commonly-used acceleration units are ft/s^2 and m/s^2. An acceleration of 1 m/s^2 means that the velocity increases by 1 m/s in 1 s.

Example C shows the use of Equation 3 in determining acceleration.

EXAMPLE C: ACCELERATION OF A CAR

Given: A driver increases the speed of a car uniformly from 35 miles per hour to 55 miles per hour in 15 seconds.

Find: Acceleration of the car.



Solution:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{55 \text{ mi/h} - 35 \text{ mi/h}}{15 \text{ s}}$$

$$= \frac{20 \text{ mi/h}}{15 \text{ s}}$$

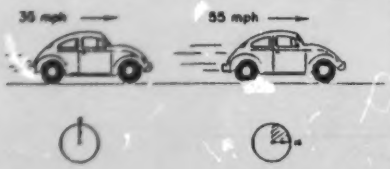
$$a = 1.34 \frac{\text{mi/h}}{\text{s}}$$

The car was accelerated 1.34 mi/h every second.

Units shown in Example C may look strange because two different time units appear. The units often are reduced in order that only one time unit is used. To reduce units in Example C in order that only one time unit is used, the unit for speed is converted from miles per hour to feet per second. Example D shows this alternate solution to Example C.

EXAMPLE D: ALTERNATE SOLUTION FOR EXAMPLE C

Given: A driver increases the speed of a car uniformly from 35 miles per hour to 55 miles per hour in 15 seconds.



Find: Acceleration of the car.

Solution:

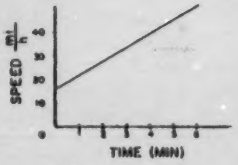
$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{(55 - 35)\ \text{mi/h}\ (\frac{44\ \text{ft/s}}{30\ \text{mi/h}})}{15\ \text{s}}$$

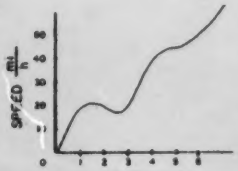
$$= \frac{29.3\ \text{ft/s}}{15\ \text{s}}$$

$$a = 1.96\ \frac{\text{ft/s}}{\text{s}}$$

The units $\frac{\text{ft}}{\text{s}}$ $\quad \text{s} = \frac{\text{ft}}{\text{s}} = \frac{\text{ft}}{\text{s}^2}$



(a)



(b)

FIG. 1

Acceleration is illustrated better by the graphs in Figure 1, where speed is recorded on the vertical axis, and where time is recorded on the horizontal axis.

A uniform change in speed, indicated on the graph by a straight line, means that linear acceleration is constant. A nonuniform change in speed, indicated on the graph by a curving line, means that linear acceleration is varying. The concepts of speed and acceleration are illustrated further in Figure 2, which represents the changing speed of a car during a short trip.

Section A of the graph shows acceleration of the car as it starts from rest and increases its speed to approximately 10 mi/h. Since the graph rises very rapidly, rate of

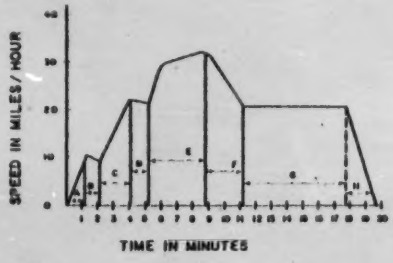


FIG. 2

increase in speed (acceleration) is large. The car in in low gear. Section B represents a slowing down (deceleration) of the car as its gears are shifted from low to second. Section C represents acceleration as the car increases its speed while in second gear. Section D again represents deceleration as gears are shifted from second to third. Section E at first represents an increasing speed and thus an acceleration. But notice that acceleration during Section E does not remain constant. The graph shows clearly that, after the initial high acceleration, the car accelerates less rapidly. Section F shows a rapid deceleration, which must result from a braking action on the part of the driver. Section G shows no acceleration, because the speed does not change during this time. Section H represents a final deceleration, which brings the car to a stop.

Figure 2 indicates clearly both accelerations and decelerations. "Acceleration" is the term generally applied to a rate of increase of speed, and "deceleration" is the term applied to a rate of decrease of speed. If a moving object is decelerating, the value for change of speed in Equation 3 will be negative, giving an overall negative value for acceleration. Thus, deceleration also is called "negative acceleration."

Acceleration is produced only when an unbalanced force sufficient to cause a change in rate of motion is applied to an object. Amount of acceleration produced depends upon both magnitude of the unbalanced force and mass of the object. The greater the unbalanced force on a given object, the greater the acceleration will be. Also, the more massive an object, the harder it is to accelerate. Furthermore, the object always will accelerate in the direction of the unbalanced force. These physical relationships (referred to as Newton's second law of motion) are stated in Equation 4.

$$a = \frac{F}{m} \qquad (4)$$

where

a = acceleration.
F = unbalanced force.
m = mass of object.

This equation often is stated in the form —

$$F = ma$$

To apply this equation, force and mass must be used in correct units. For example, when mass is measured in kilograms and force is measured in newtons ($\frac{kg \cdot m}{s^2}$), acceleration will be in m/s^2. Use of the acceleration formula is illustrated in Examples E and F.

EXAMPLE E: ACCELERATION OF A MASS

Given: An unbalanced force of 35 N is applied to a 7-kg object.

Find: Acceleration produced (neglect friction).

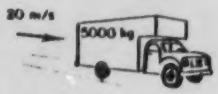
Solution: $a = \frac{F}{m}$

$$= \frac{35\ N}{7\ kg} = \frac{35\ \frac{kg \cdot m}{s^2}}{7\ kg}$$

$$a = 5\ m/s^2$$

EXAMPLE F: TRUCK DECELERATION

Given: A 5000-kg truck has a speed of 20 m/s.



Find: Braking force required to stop in 60 seconds.

Solution: First, find rate of deceleration:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{0 - 20\ m/s}{60\ s}$$

$$= \frac{-20\ m/s\ (1/s)}{60\ s/1\ (1/s)}$$

$$a = -0.33\ m/s^2$$

Now, find force required:

$$F = ma$$

$$= 5000\ kg \cdot -0.33\ m/s^2$$

$$F = -1650\ \frac{kg \cdot m}{s^2} = -1650\ N$$

A force of 1650 N is required to stop the truck in the time indicated. The negative sign tells that braking force was applied in a direction opposite to motion of the truck.

Acceleration of a body due to earth's gravity is 32 ft/s^2 or 9.8 m/s^2. This means that a freely-falling body is increasing its velocity an additional 32 ft/s for every second that it falls (assuming that there is no wind resistance). After two seconds, a falling body that started from rest will be falling with a velocity of 64 ft/s, and after five seconds, with a velocity of 160 ft/s. Weight of a body is a measure of the pull of gravity of the mass of that body. This relationship is shown in Equation 5.

$$W = mg \tag{5}$$

where
W = weight (N or lb).
m = mass (kg or slugs).
g = acceleration of gravity.

The importance of using weight and mass correctly is shown by Example 6.

EXAMPLE 6: TOWING AN AUTOMOBILE

Given: A 3700-lb automobile is being towed on a level road by a towtruck accelerating at 3.8 ft/s^2.

Find: Tension in the towline (force required).

Solution: $F = ma$

Wrong Solution:

F (tension) = (370 lb)(3.8 ft/s^2)

$$F = 14{,}060 \ \frac{lb \cdot ft}{s^2}$$

A tention of $\frac{lb \cdot ft}{s^2}$ does not make sense. Tension should be in pounds! What went wrong?

The mistake is that 3700 lb is not the mass but the weight of the car. Therefore, this solution is incorrect.

Correction Solution:

First, find mass of the car being accelerated:

$W = mg$

$m = \frac{W}{g}$

$$m = \frac{3700 \text{ lb}}{32 \text{ ft/s}^2}$$

Recall that $\frac{\text{lb}\cdot\text{s}^2}{\text{ft}} = \text{slugs}$

$$m = 115.6 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}} = 115.6 \text{ slugs}$$

Now, find tension on the rope:

$$F = ma$$
$$= (115.6 \text{ slugs})(3.8 \text{ ft/s}^2)$$
$$= \left(115.6 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)\left(3.8 \frac{\text{ft}}{\text{s}^2}\right)$$
$$F = 439 \text{ lb}$$

Tension in the towline is 439 lb.

MECHANICAL ROTATIONAL RATES

Rate of rotational motion is called angular velocity. Angular velocity is defined as the rate of change of an angle with time. Common units for measuring angular velocity are revolutions per minute (rpm or r/min) and radians per second (rad/s).

Recall from Module 2-0, "Work," that 1 revolution equals 2π radians. The relationship between rpm and rad/s is illustrated in Example H.

EXAMPLE H: UNITS FOR ANGULAR VELOCITY

Given: A phonograph turntable has an angular velocity of 45 rpm.

Find: The angular velocity in rad/s.

Solution: Angular velocity = $(45 \frac{\text{rev}}{\text{min}})(\frac{2\pi \text{ rad}}{1 \text{ rev}})(\frac{1 \text{ min}}{60 \text{ s}})$

Angular velocity = 4.7 rad/s.

The angular velocity is 4.7 rad/s.

In the rotational mechanical system, the basic rate equation is Equation 6.

$$\omega = \frac{\theta}{t} \qquad (6)$$

where

ω = angular velocity.
θ = angular displacement.
t = elapsed time.

The use of this equation in determining angular velocity is illustrated in Example I.

EXAMPLE I ANGULAR VELOCITY

Given: A grindstone rotates 600 revolutions in 5 minutes.

Find: Angular velocity.

Solution:

$$\omega = \frac{\theta}{t}$$

$$= \frac{6000 \text{ revolutions}}{5 \text{ minutes}}$$

$$= 120 \text{ rpm}$$

$$= 120 \left(\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$= 120 \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$= 4\pi \text{ rad/s}$$

$$\omega = 12.56 \text{ rad/s}$$

When an unbalanced torque is applied to a rotary system, the angular velocity of that system is changed. The rate at which this change takes place is called "angular acceleration." Angular acceleration can be determined by dividing change in angular velocity by time required for this change. The formula for calculating angular acceleration is given in Equation 7.

$$\alpha = \frac{\omega_2 - \omega_1}{t} \qquad (7)$$

where

α = angular acceleration.
ω_1 = initial angular velocity (before change).
ω_2 = final angular velocity (after change).
t = time in which the change takes place.

Example J shows how the formula can be used to solve a problem.

EXAMPLE J: ANGULAR ACCELERATION

Given: A brake is applied to a flywheel for 5 seconds, reducing angular velocity of the flywheel from 2.2 rad/s to 1.8 rad/s.

Find: Angular acceleration produced.

Solution:

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$= \frac{1.8 \text{ rad/s} - 2.2 \text{ rad/s}}{5 \text{ s}}$$

$$= \frac{-0.4 \text{ rad/s}}{5 \text{ s}}$$

$$\alpha = 0.08 \text{ rad/s}^2$$

Angular acceleration produced is -0.8 rad/s^2. The negative sign indicates this is a deceleration of 0.08 rad/s^2.

FLUIDAL SYSTEMS

The rate of flow of fluids is often an important consideration in hydraulic and pneumatic systems. However, a measure of fluid velocity is not in itself adequate to describe flow rate, since the volume or capacity of fluid moving through a duct or pipe must be considered. Thus, one measures volume per second, rather than simply distance per second. For example, if the velocity of fluid in two pipes is the same, less fluid will flow through a small pipe in the same time, as illustrated in Figure 3. The volume of fluid that flows in a given time is much more important. This quantity is called the "volume flow rate." It is found by dividing a measured volume by the time required for this volume to flow from a pipe. Equation 8 shows the formula for calculating volume flow rate. Note that the displacementlike quantity in this instance becomes the volume of fluid moving past a certain position in the elapsed time.

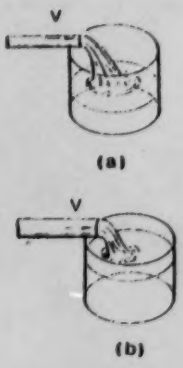


FIG. 3

$$Q_V = \frac{V}{t} \qquad (8)$$

where

Q_V = volume flow rate.
V = volume of fluid displaced.
t = elapsed time.

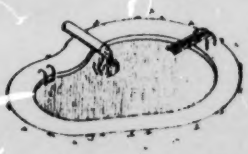
Units of volume flow rate could be any volume unit divided by any time unit. Examples include the following: m^3/s, ft^3/s, gal/min, liters/s, cm^3/s. Example K illustrates how volume flow

rate is calculated.

EXAMPLE K: VOLUME FLOW RATE

Given: A swimming pool holds 10,000 gallons; 2 hours are needed to fill pool.

Find: Volume flow rate of water through the hose.



Solution:

$$Q_V = \frac{V}{t}$$

$$= \frac{10{,}000 \text{ gal}}{(2\text{h})\left(\frac{60 \text{ min}}{1 \text{ h}}\right)}$$

$$Q_V = 83 \text{ gal/min}$$

For some purposes (especially for gases), the desirability arises to consider mass flow rate rather than volume flow rate. Mass flow rate is defined as "the fluid mass moving past a given position in a given amount of time." Equation 9 defines mass flow rate. Note that here the displacementlike quantity becomes the mass moving past a fixed position in the elapsed time.

$$Q_m = \frac{m}{t} \tag{9}$$

where

Q_m = mass flow rate.
m = mass of fluid displaced.
t = elapsed time.

Example L shows how flow rate can be calculated.

EXAMPLE L: MASS FLOW RATE

Given: 180 kg of ammonia gas flowing through the cooling coils of a refrigerated truck in 6 minutes.

Find: Mass flow rate of the gas.

Solution:

$$Q_m = \frac{m}{t}$$

$$= \frac{180{,}000 \text{ g}}{6 \text{ min}}$$

Q_m = 30,000 g/min = 30 kg/min

The ammonia gas is flowing at a rate or 30,000 grams/minute of 3 kg/min.

ELECTRICAL SYSTEMS

In electrical systems the displacementlike quantity is charge. Recall from Module 1-0, "Force," that like charges attract and unlike charges repel. This characteristic of electrical charge results in a tendency for positive and negative charges to be distributed as evenly as possible in an electrical system. If the system contains a concentration of positive charge in one volume and a concentration of negative charge in another, a potential difference (forcelike quantity) is present. If charges are free to move, the potential difference will cause charge to flow until the system is in electrical equilibrium. Electric current is the rate of charge flow and is defined by Equation 10.

$$I = \frac{Q}{t} \qquad (10)$$

where

I = electric current.
Q = charge transferred.
t = elapsed time.

Current is measured in amperes (amp or A). One ampere is equal to one coulomb of charge flowing each second. In metallic electrical conductors the moving charge is composed of electrons that flow through the metal. There are 6.25×10^{18} electrons in one coulomb. Thus, a current of one amp represents a transfer of 6.25×10^{9} electrons from the point of negative potential to the point of positive potential every second. Example M illustrates the use of Equation 10 in determining the charge transferred in a household circuit.

EXAMPLE M: ELECTRICAL CURRENT

Given: A home electrical circuit is limited to 15 amps by a fuse or circuit breaker.

Find: How many electrons have flowed through the circuit in 10 seconds when it has a maximum load.

Solution: A current of 15 amps is equivalent to a flow rate of

15 coulombs/seconds.

$I = \frac{Q}{t}$

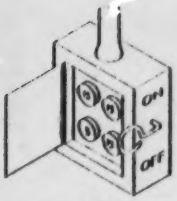
$Q = It$

$= (15\ C/s)(10\ s)$

$= 150\ C$

$= (150\ C)\left(\frac{6.25 \times 10^{18}\ \text{electrons}}{1\ C}\right)$

$Q = 9.4 \times 10^{20}$ electrons



FUSE BOX

Alternating current (a.c.) and direct current (d.c.) are terms that refer to the direction in which electrical charge moves. Direct current means that electrical charge flows in one direction only, even though the amount of current may vary. In alternating current, the direction of the current changes (alternates) so that the charge flows back and forth. Commercial power companies generate electric current that changes direction at the rate of 120 times each second. (The popularity of a.c. arises from its ability to transmit electrical energy great distances without large heat losses in the wires. One advantage in the use of a.c. electricity in our homes is that a smaller wire will carry more current.)

THERMAL SYSTEMS

In any system where a temperature difference exists, heat moves from the high-temperature region to the low-temperature region. Movement in this system, however, is different from movement in the other energy systems discussed in this module. In other systems, an object of some kind moves from one location to another; in thermal systems, the quantity that moves is heat energy. This energy is contained in a substance in the form of vibrational energy of the molecules. A higher temperature indicates a higher vibrational rate of the molecules and thus a greater quantity of heat energy contained in the substance.

In any substance, heat energy is transferred when an energetic vibrating molecule bumps a neighboring molecule with a lower vibrational rate. The more-energetic molecules loses some of its thermal energy and its vibrational rate decreases; the less-energetic molecule gains this energy and its vibrational rate increases. This process continues in a closed system until the thermal energy is evenly distributed and there is no longer a temperature difference.

Heat flow rate is the amount of heat energy flowing through a surface in a given amount of time. The formula used to calculate heat flow rate is given in Equation 11. Here the displacementlike

quantity is less clear than distance, volume or mass. In thermal systems, it becomes a measure of the heat energy flow past a fixed position in the time elapsed.

$$Q_H = \frac{H}{t} \qquad (11)$$

where

Q_H = heat flow rate.
H = heat energy transferred.
t = elapsed time.

Designers of heating and cooling systems are vitally concernad with rate of heat flow. Manufacturers' specifications for heating and cooling systems, however, sometimes are misleading, because they are given in units of Btu's. This terminology is misleading, because what the manufacturer means by "Btu's" is actually "Btu's per hour," which is a heat flow rate. For instance, a 10,000-Btu air conditioner is capable of removing room heat at a rate of 10,000 Btu's every hour. Example H shows how Equation 11 is used to solve problems involving heat flow rate.

EXAMPLE H: HEAT FLOW RATE

<u>Given:</u> A 5000-Btu tent heater runs continuously for 5 hours and 30 minutes.

<u>Find:</u> Amount of energy produced.

<u>Solution:</u> The 5000-Btu specification means a heat flow rate of 5000 Btu/hour.

$$Q_H = \frac{H}{t}$$

$$H = Q_H t$$

$$= (5000 \text{ Btu/h})(5\ 1/2 \text{ h})$$

$$H = 27{,}500 \text{ Btu}$$

SUMMARY

The concept of rate is important in all energy systems. Any change in a system takes place in a certain elapsed time. Rate is the ratio of change in a physical quantity to the time required for that change. This module has dealt with the basic rates in four energy systems — mechanical, fluidal, electrical, and thermal. Many other rates commonly are used in science and technology. Some of these are discussed in application modules. One rate is so important that it is considered to be a separate unifying concept. Power is the rate of energy transfer and will be discussed in detail in Module 6-0, "Power."

Table 2 summarizes the basic rates presented in this module.

TABLE 2: RATE AS A UNIFYING CONCEPT

ENERGY SYSTEM	RATE = $\frac{\text{DISPLACEMENTLIKE QUANTITY}}{\text{ELAPSED TIME}}$
MECHANICAL	
Translational	Velocity = $\frac{\text{Displacement}}{\text{Elapsed Time}}$
Rotational	Angular Velocity = $\frac{\text{Angular Displacement}}{\text{Elapsed Time}}$
FLUIDAL	Volume Flow Rate = $\frac{\text{Volume Displaced}}{\text{Elapsed Time}}$
	Mass Flow Rate = $\frac{\text{Mass Displaced}}{\text{Elapsed Time}}$
ELECTRICAL	Current = $\frac{\text{Charge Transferred}}{\text{Elapsed Time}}$
THERMAL	Heat Flow Rate = $\frac{\text{Heat Energy Transferred}}{\text{Elapsed Time}}$

MECHANICAL SYSTEMS

MATERIALS

Toy car, either wind-up or battery powered.
2-meter track for car.

PROCEDURE

The student is to use a wind-up or a battery powered toy car to calculate velocity of the car and speed with which the wheels turn.

1. Mark "start" and "finish" lines on a straight course at least two meters long.
2. Start car a reasonable distance in front of "start" line (to allow speed to become steady); time car carefully as it travels from "start" to "finish" line. Record this time in Data Table 1.
3. Calculate average speed of car in m/s, ft/s and mi/h.
4. Measure wheel diameter on car and list in Data Table 1.
5. Calculate wheel circumference, $c = \pi d$.
6. Divide raceway length by wheel circumference to determine how many times wheel rotated while it traveled across raceway.
7. Calculate angular velocity of wheel in rpm and in rad/s.

DATA TABLE 1	
Raceway length _____	
Time from start to finish _____	
Average speed (calculated) _____	m/s
_____	ft/s
_____	mi/h

Wheel diameter _____ mm

Wheel circumference (c = _____ mm

Number of rotations to cover raceway _____ revolutions

Angular velocity of wheel (calculated) _____ rpm

_____ rad/s

FLUID SYSTEMS

MATERIALS

Gallon or liter container.
Meter stick.
Stopwatch or timer.
Balance.

PROCEDURE

Several cooling systems require a specified volume flow rate for adequate operation. Specified values may be given in units of gallons per minute or liters per minute. In this experiment, a water faucet is used to simulate a discharge pipe from a cooling system and to calculate volume flow rate coming from the faucet (Figure 4).

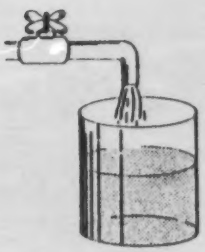


FIG. 4

1. Collect flowing water in a container such as a gallon or liter container; accurately measure time required to fill container.

2. Record values in Data Table 2. From these measurements, calculate volume flow rate in both gallons per minute and in liters per minute. Enter values in Data Table 2.

3. Using a balance scale, determine mass of water in container.

4. Calculate mass flow rate in kilograms per minute and in slugs per minute. Enter values in Data Table 2.

DATA TABLE 2	
Container volume	_____
Time to fill container	_____
Mass of water in container	_____
Volume flow rate	_____ gal/min
	_____ l/min
Mass flow rate	_____ kg/min
	_____ slugs/min

THERMAL SYSTEMS

MATERIALS

Conductometer (Sargent Welch #1653 or equivalent).
Bunsen burner or candle.
Paraffin.

PROCEDURE

Heat flows more rapidly through some materials than through others. This experiment determines which of the materials tested allows the greatest heat flow rate. The instrument used to make these determinations is the "Conductometer," a device having several spokes radiating outward from the central hub, as illustrated in Figure 5.

1. Place a piece of melted paraffin on end of each spoke; allow it to cool.

2. Place a lighted bunsen burner (or candle) under central hub. Observe which spokes have most rapid head flow by noting the order in which paraffin pieces melt. List the metals in order, from most rapid head flow rate to slowest heat flow rate. (1) _____ (2) _____ (3) _____ (4) _____ (5) _____ (6) _____.

FIG. 5

MATERIALS

6- or 12-volt battery or d.c. power supply.
6- or 12-volt d.c. bulb with mounting base and connecting wires.
d.c. ammeter 0- to 2-amp.

PROCEDURE

Rate of flow of electric charge (electric current) is measured with an ammeter. Place ammeter in an electric circuit along with a battery and a small d.c. light bulb, according to Figure 6. How much electric current is recorded by milliammeter when the bulb is lit?

Current = _____.

How many coulombs/s is this? _____.

If the light bulb burns for one hour, how many electrons have moved through bulb filament?

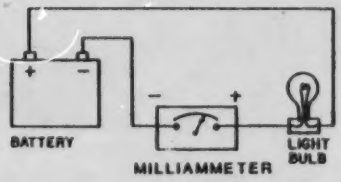


FIG. 6

SELF CHECK

1. Define the following rates, and express their basic units both in the SI and the English system of units:

   Linear speed.
   Linear acceleration.
   Angular velocity.
   Angular acceleration.
   Volume flow rate.
   Mass flow rate.
   Electrical current.
   Heat flow rate.

2. Calculate an airplane's average speed if it travels 3200

kilometers in 3.5 hours.

3. If a car traveling at 55 miles per hour stops in 16.3 seconds, calculate the car's deceleration in ft/s^2.

4. If a driver averages 55 mi/h, how much time will be required to make a 250-mile trip?

5. A force of 200 N accelerates an object with a mass of 100 kg. If the object was initially at rest, determine its speed after 3 seconds.

6. A flywheel of radius 60 cm initially at rest requires 25 seconds to reach an angular velocity of 1750 r/min. Calculate angular acceleration of the flywheel.

7. A motor rotates at a rate of 5000 rpm. How much time is required for one complete revolution?

8. A gas pump moves gasoline from an underground storage tank to the tank in a car at the rate of 15 liters per minute. Calculate time required to fill a 20-gallon gas tank with this pump.

9. If an air conditioner is rated at 10,000 Btu's, calculate how much heat energy it will remove from a room in one day if it operates 20% of the time.

10. Electric current flows at the rate of 0.5 amps through a normal 60-watt light bulb. Calculate how many coulombs of electric charge this represents in 12 hours of continuous operation.

11. State the general equation for rate; and explain how it applies to mechanical, fluidal, electrical, and thermal energy systems.

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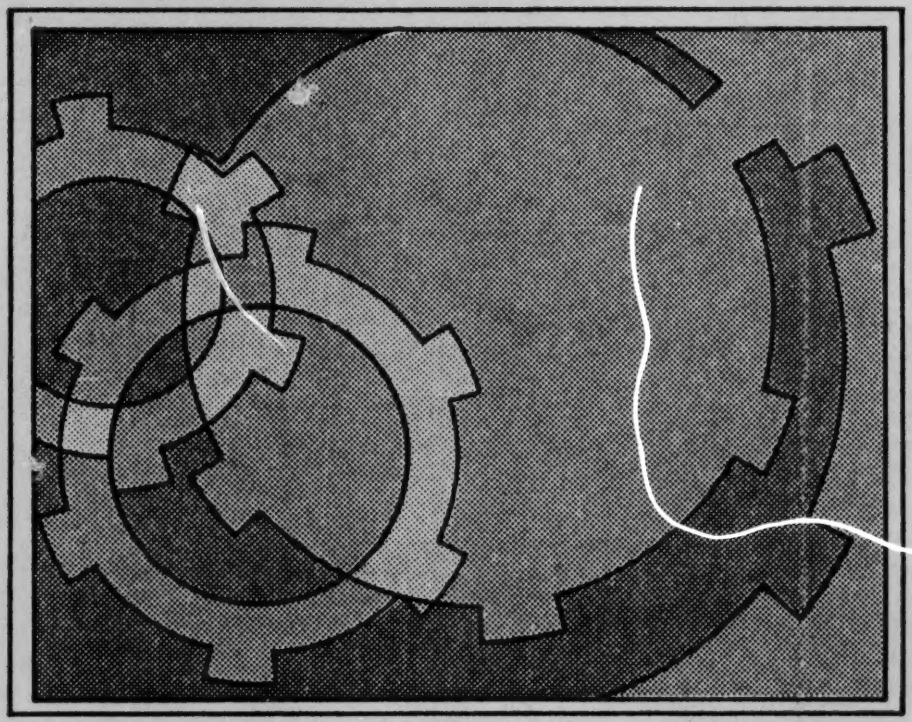
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 1

GRAPHIC APPROACH TO LINEAR MOTION



MECHANICAL APPLICATION OF RATE

Technical Education Research Center-Southwest

TERC-SW

INTRODUCTION

A car is cruising down the freeway at 55 miles per hour. One hundred yards behind, a faster car is closing in at 70 miles per hour in the same lane. The driver of the faster car slams on the brakes. With the brakes locked, the car will need 150 yards to stop. Will there be a crash? In other words, the question is, will these two cars ever be in the same place at the same time? To answer such questions it is necessary to develop general connections between speeds, positions and times.

The general subject of relationships between speed, position and time is called kinematics. In this module, only the description of motion along a straight line will be studied. Such motion is called rectilinear motion. A later module will concern present circular motion, which is somewhat more complicated.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Displacement.
  - Velocity.
  - Path of motion.
  - Elapsed time.
  - Velocity diagram.
  - Area under velocity versus time curves.
- Given the velocities of an object and the elapsed time for each velocity, draw a velocity diagram and determine the

displacement during each interval and the total displacement.

- Measure and record the positions and velocities of an automobile during constant acceleration and constant velocity; draw a velocity diagram for the motion of the car and determine:
  - Displacement during constant accleration.
  - Displacement during constant velocity.
  - Total displacement.

POSITION AND DISPLACEMENT

To determine velocity of an object, it is necessary to measure the position of the object at different times. To describe the position from which the measurements are to be made, an origin or landmark must be specified. Distances and direction from this origin must also be stated. Thus, for example, when the position of a car is given as being 3 miles east of this classroom on the interstate highway, the location of the car is completely known. Position of the car would not be completely specified by saying simply that it was 3 miles away. It would not be known if it were 3 miles north or 3 miles south.

Because only motion along a straight line is to be considered in this module, a relatively simple coordinate system is required. This coordinate system is shown in Figure 1.

The origin of this system will be the center of the line. All distances to the right side are positive; all to the left side are negative. It is then possible to specify any position on the line by a positive or negative number that gives both the distance (in some convenient units) and the direction of that point from the origin. Such signed numbers will be referred to as the coordinate of the point. For example, the coordinates X_1, X_2 and X_3 are located at +3, +6 and -7.

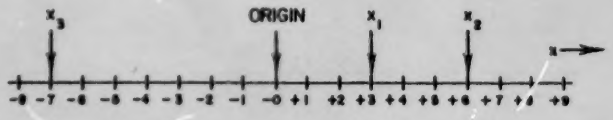


Fig. 1

The "path of motion" is the line traced by the moving particle in a given interval of time. This path may be a straight line (rectilinear motion) or a plane curve (plane curvilinear motion), or a space curve (spatial curvilinear motion). In this module only the first case will be considered and therefore the path of motion will always be a straight line. As a car or object moves along its path of motion its position is constantly changing. The change in position of a moving object is called "displacement." If an object moves from position X_1 to position X_2, the displacement d is given by $X_2 - X_1$. Displacement is positive when X_2 is greater then X_1, negative when X_2 is less than X_1. As is shown in Figure 2, whether the displacement is positive or negative depends only on the direction of movement and does not depend upon which side of the origin the change

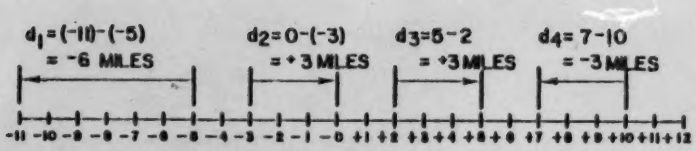


Fig. 2

takes place. The arrows show direction of the displacements. Displacements d_2 and d_3 are in a positive direction and are equal in magnitude. Displacements d_1 and d_4 are in the negative direction, and d_1 is twice d_4.

VELOCITY

The quantity that tells how fast and in what direction something moves is called "velocity." The velocity of an object moving at a constant rate along a straight line is defined as the displacement per unit time. Stated analytically, velocity is defined as

$$v = \frac{X_2 - X_1}{t_2 - t_1} = \frac{d}{t} \qquad (1)$$

where $t = t_2 - t_1$ is the time interval during which the displacement occurred and is commonly referred to as "elapsed time." Because time can only move in the forward direction (t is always positive), the sign of the velocity is determined by the sign of the displacement. Thus, when displacement is positive, motion is in the positive direction and velocity is positive regardless of where it takes place on the coordinate line. Conversely, when displacement is negative, velocity is negative and motion is in the negative direction.

As an example, suppose that displacement d_1 in Figure 2 took place in 0.1 s. Velocity for this displacement and elapsed time would be:

$$v_1 = \frac{d_1}{t} = \frac{-6 \text{ cm}}{0.1 \text{ s}} = -60 \frac{\text{cm}}{\text{s}}.$$

If the displacement d_2 took place in 0.6 s,

$$v_2 = \frac{d_2}{t} = \frac{+3\ cm}{0.6\ s} = +5.0\ cm/s.$$

Note that the signs, numbers and units must be carried throughout these calculations to ensure the correct answer.

VELOCITY DIAGRAM

Velocity of an object is often plotted as a function of time in graphical form. Such a graph is called "velocity diagram." Figure 3 shows three such velocity diagrams.

Figure 3a shows a graph of an object moving in the positive direction at a constant velocity of +1.6 m/s. Figure 3b shows the graph of an object moving in the negative direction at a constant velocity of -1.6 m/s. Figure 3c shows the graph of an object moving at various velocities, both in the positive and negative directions.

Applying this graphical method to a more complicated motion will illustrate all of the concepts studied thus far. Suppose that the motion of a car along a straight road is represented by the data in Table I.

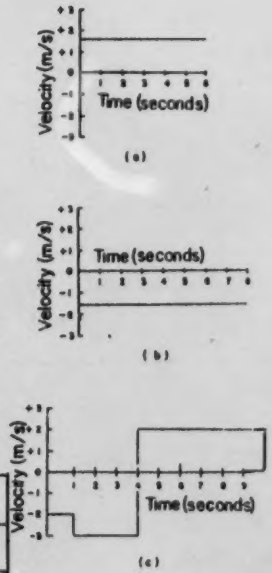


Fig. 3

TABLE 1. MOTION OF A CAR WITH A VARIABLE VELOCITY

Time Interval Number	Duration Of Interval	Velocity During Interval	Displacement In Units
a	0.40 h	40 mi/h	16.0
b	0.10 h	20 mi/h	2.0
c	0.30 h	30 mi/h	9.0
d	0.50 h	75 mi/h	37.5
e	0.10 h	25 mi/h	2.5

To compute the displacement for each of these time intervals, Equation 1 is rearranged to give:

$$d = vt. \qquad (2)$$

Thus, for time interval "a,"

$$d = 40 \text{ mi/h} \times 0.40 \text{ h}$$
$$d = 16 \text{ mi.}$$

The velocity diagram of the data in Table I is shown in Figure 4.

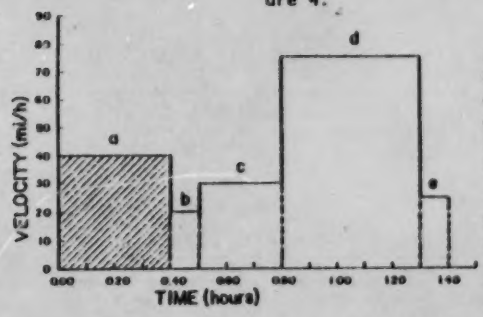


Fig. 4

Such sudden jumps in velocity are not realistic, but for simplicity we shall assume that the car can change speed in negligible time. By looking at the graphical presentation in Figure 4, it is easy to see when the car changes speed, as well as its direction of motion.

It is also possible to determine graphically the displacement during each of the five time intervals. During any of the intervals, the position changes by an amount determined by Equation 2. For example, consider time interval "a" where:

$$d = vt = 40 \text{ mi/h} \times 0.40 \text{ h} = 16 \text{ mi.}$$

Using this equation is equivalent to finding the area under the velocity curve. In any interval, the height of the graph gives velocity during the interval and the horizontal length gives the time. Thus v times t is the height times the base, or the "area" of the rectangle. As an illustration, consider the shaded area in Figure 4. The area of this rectangle is:

$$\text{area} = \text{height} \times \text{base}$$
$$= 40 \text{ mi/h} \times 0.40 \text{ h}$$
$$\text{area} = 16 \text{ mi.}$$

Thus, finding the rectangular area under the velocity diagram gives the displacement during that interval and is equivalent to the analytical solution, using Equation 2.

The total displacement of the car in 1.40 h is obtained by adding up the areas of all the intervals in Figure 4. As an exercise, determine the sum of the areas in Figure 4 and see if

it agrees with the sum of the displacements listed in Table 1. Either method should give 67 miles traveled.

How are displacements determined if both negative and positive velocities occur as in Figure 3c? During the first four seconds, displacement is the sum of the areas of the first two rectangles (the time-axis always determines one side of the rectangle).

$$d = d_1 + d_2 = (-2 \text{ cm/s})(1 \text{ s}) + (-3 \text{ cm/s})(3 \text{ s})$$
$$d = -11 \text{ cm}.$$

During the next 6 seconds, the object had a positive velocity, back toward the origin, and its displacement was positive.

$$d = d_3 = +2 \text{ cm/s} \times 6 \text{ s}$$
$$d = +12 \text{ cm}.$$

The total displacement between 0 and 10 seconds is:

$$d = -11 \text{ cm} + 12 \text{ cm}$$
$$d = 1 \text{ cm}.$$

After having traveled 11 cm to the left and then 12 cm to the right, this object ended up only 1 cm to the right of where it started. This illustrates the importance of not confusing displacement during a time interval with total distance traveled during that interval. In this case, the object was displaced 1 cm from its starting point after having traveled a total of 23 cm. As another example, astronauts traveled 480,000 miles to the moon and back, yet, because they ended up where they started their displacement was 0. Thus, in kinematics, displacement and total distance traveled are not the same.

VARYING VELOCITY

If an object such as a car travels with a constantly varying velocity, the methods outlined previously still apply, but are more complicated. For example, Figure 5 shows the velocity versus time graph of a car that is changing its velocity during part of its trip. During the first 0.3 hour, the car changes

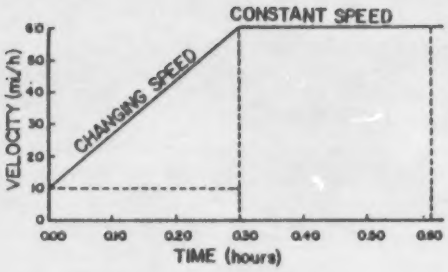


Fig. 5

speed smoothly from 10 mi/h to 60 mi/h. During the next 0.30 hour, the car travels at a constant speed of 60 mi/h.

The area under the curve is still the displacement. During the first 0.30 hour, the area can be calculated from the sum of the areas of the rectangle and the triangle.

$$d = (10 \text{ mi/h} \times 0.30 \text{ h}) + (\tfrac{1}{2} \times 50 \text{ mi/h} \times 0.30 \text{ h})$$
$$= 3 \text{ mi} + 7.5 \text{ mi}$$
$$d = 10.5 \text{ mi}.$$

For the next 0.30 hour, the car was travelling at a constant velocity of 60 mi/h and its displacement for this time interval was:

$$d = 60 \text{ mi/h} \times 0.30 \text{ h}$$
$$d = 18 \text{ mi}.$$

The total displacement for the 0.60 h trip was:

$$d = 10.5 \text{ mi} + 18 \text{ mi}$$
$$d = 28.5 \text{ mi}.$$

MATERIALS

Stopwatch or wristwatch.
Car.
Tape measure.

PROCEDURE

1. Have a timekeeper and an observer accompany driver.
2. Start car from a marked spot (position 1) on a straight, level, somewhat isolated road. Note odometer reading.
3. Accelerate car at a smooth and constant rate until a velocity of 30 mi/h is achieved (position 2).
4. Timer will record elapsed time between when car started to move and when car reached velocity of 30 mi/h. Timer will also note the odometer reading and position of car at this point.
5. Driver will then maintain a constant velocity of 30 mi/hr for 6 seconds to point 3.
6. Timer will record elapsed time for this constant velocity portion of trip; note odometer reading and position.
7. Driver, timer and observer will return to start; measure the distance to point where velocity of 30 mi/h was first achieved and distance from this point to position of car at end of 6-second interval. This is a check on odometer readings taken above in 1, 4 and 6. Be sure to read odometer to nearest tenths or better.
8. All data will be recorded in data table.
9. Using the data in data table, plot a velocity diagram similar to that shown in Figure 6.
10. Calculate displacement for both time intervals; compare it with results measured in Figure 6.

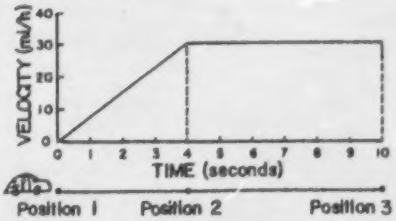


Fig. 6

$$d = \frac{1}{2}(30\ \text{mi/h})(4\ \text{s})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) + (30\ \text{mi/h})(6\ \text{s})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)$$

$$= 0.0167\ \text{mi} + 0.05\ \text{mi} = 0.0667\ \text{mi}$$

d = 0.0667 mi = 352.2 feet = total distance traveled.

DATA TABLE		
Position 1	Position 2	Position 3
$t = 0$ $v = 0$ $d = 0$	t_1 = $v = 30$ mi/h d_1 =	$t_2 = t_1 + 6$ s $v = 30$ mi/h d_2 =
Total displacement = $d_1 + d_2$ _____. Total displacement calculated = _____. Area under velocity diagram = _____.		

SELF CHECK

1. Define or explain:
   - Displacement.
   - Velocity.
   - Path of motion.
   - Elapsed time.
   - Velocity diagram.
   - Area under velocity diagram.
2. Data for a trip made in a car is given in the accompanying table:

Time interval	Duration of interval (h)	Velocity (mi/h)
1	0.20	30
2	0.40	55
3	0.10	20

How far does the car go in the first interval?
What is the displacement for the trip?
Make a velocity versus time graph for this trip.
How long does it take the car to travel the first 20 miles?
Indicate the area on graph that corresponds to the first 20 miles of travel.

3. Car A is stopped at a traffic light. The light turns green, and A starts up. Just as it does so, car B passes it going at a steady velocity. Their velocity diagrams are shown in Figure 7.
   How long does it take car A to reach the speed of car B?
   At that time, how much is car B ahead of car A?
   Which car is ahead, and by how much, at the end of 0.010 h?
   At what time does car A catch up with car B?
   How far have they traveled from the traffic light by the time car A catches car B?

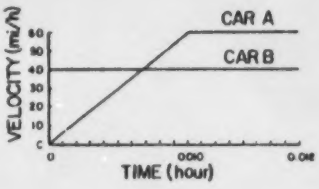


Fig. 7

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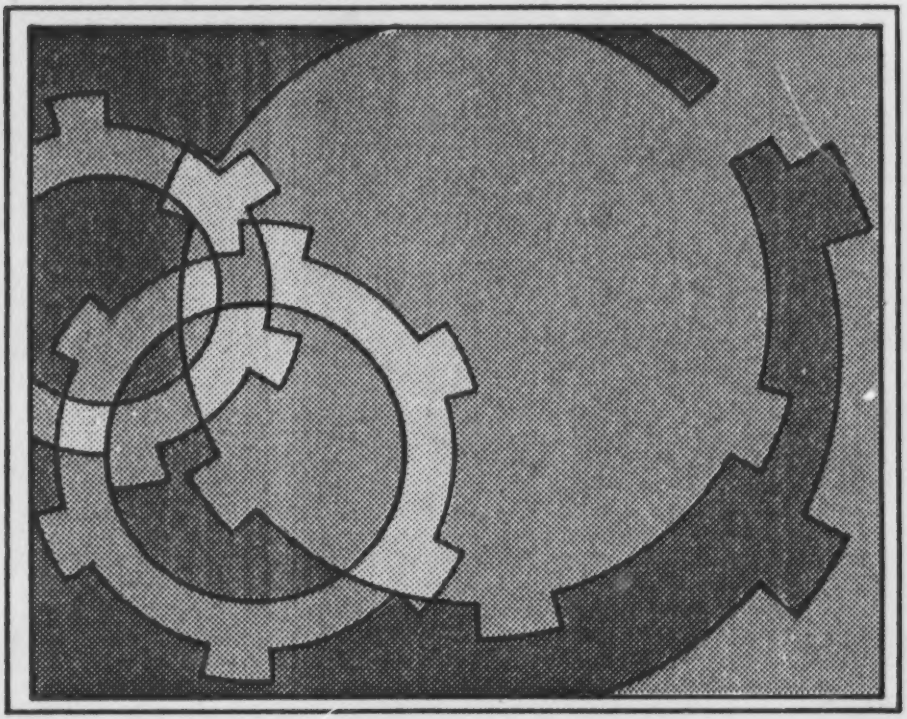
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 2

VELOCITY AT CONSTANT ACCELERATION



MECHANICAL APPLICATION OF RATE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Velocity, or speed, is the basic rate-like quantity of the mechanical system. Since linear acceleration is the rate at which speed changes, speed of an object at any later time can be determined if its initial speed and acceleration are known. In this module the student will solve problems involving linear velocity and acceleration. Since the solutions to problems involving variable acceleration involve mathematical techniques beyond the scope of this course, only constant acceleration is used. In the laboratory the student will determine acceleration by measuring the speed of an object at the beginning and at the end of a time interval.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate final speed, given initial speed, acceleration and duration of acceleration.
- Calculate acceleration, given speed at two different times.
- Calculate time required to accelerate an object from one speed to another, at a constant acceleration.
- Given the appropriate equipment, determine experimentally the acceleration of a moving body by measurements of speed and time.

Figure 1 shows the graph of speed versus time for a moving object. At time t_1 the speed was v_1, and at time t_2 it was v_2. Acceleration is the slope of the line, which can be found by Equation 1.

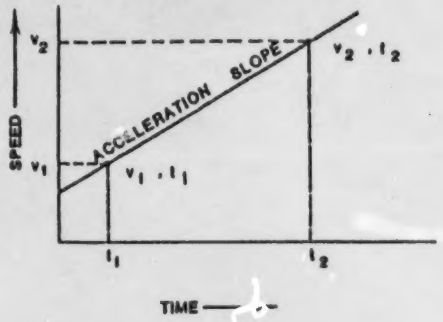


FIG. 1

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

or

$$a = \frac{v_2 - v_1}{t} \qquad (1)$$

where

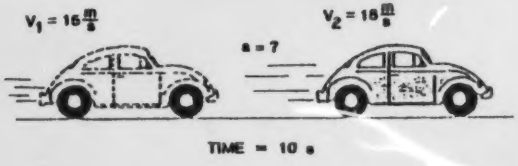
a = acceleration.
v_1 = initial speed (speed before change).
v_2 = final speed (speed after change).
t_1 = time at which speed was v_1.
t_2 = time at which speed was v_2.
t = time elapsed between the motion at v_1 and the motion at v_2.

Example A shows the use of Equation 1 in solving a problem.

EXAMPLE A: ACCELERATION OF AN AUTOMOBILE

Given: An automobile accelerates from 15 m/s to 18 m/s in 10 s.

Find: Its acceleration.



Solution:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{(18\ m/s - 15\ m/s)}{10\ s}$$

$$= \frac{3\ m/s}{10\ s}$$

$$a = 0.3\ \frac{m}{s^2}$$

The automobile was accelerated 0.3 m/s every second.

Equation 1 can be rearranged into another useful form. When both sides are multiplied by t, $a = \frac{v_2 - v_1}{t}$ becomes $at = v_2 - v_1$. Next, v_1 can be added to both sides of the equation, and the sides reversed, to produce Equation 2 as follows:

$$at + v_1 = v_2 - v_1 + v_1$$

$$at + v_1 = v_2$$

$$v_2 = v_1 + at \qquad (2)$$

where

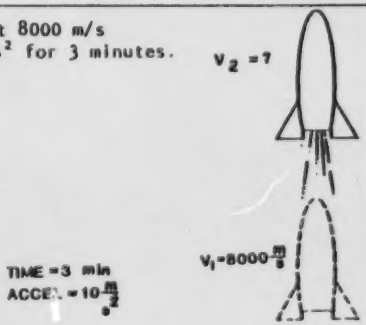
v_2 = final speed.
v_1 = initial speed.
a = acceleration.
t = time.

Example 8 demonstrates the use of Equation 2 in solving a problem.

EXAMPLE 8: FINAL SPEED OF A ROCKET

Given: A rocket traveling at 8000 m/s accelerates at 10 m/s^2 for 3 minutes.

Find: Its new speed.



Solution: $v_2 = v_1 + at$
$= 8000 \text{ m/s} + (10 \text{ m/s}^2)(180 \text{ s})$
$= 8000 \text{ m/s} + 1800 \text{ m/s}$
$v_2 = 9800 \text{ m/s}$

After 3 minutes the speed is 9800 m/s.

Another form for the acceleration formula is easily derived for time in Equation 3 as follows:

$$a = \frac{v_2 - v_1}{t}$$
$$at = v_2 - v_1$$
$$t = \frac{v_2 - v_1}{a} \qquad (3)$$

where

t = time.
v_2 = final speed.
v_1 = initial speed.
a = acceleration.

Example C shows the use of Equation 3 in solving a problem.

EXAMPLE C: TIME REQUIRED TO REACH A GIVEN SPEED

Given: An object is accelerating at 0.5 m/s^2.

Find: Time required to produce a speed of 27 m/s, beginning from rest.

Solution:
$$t = \frac{v_2 - v_1}{a}$$
$$= \frac{27 \text{ m/s} - 0 \text{ m/s}}{0.5 \text{ m/s}^2}$$
$$= \frac{27 \text{ m/s}}{0.5 \text{ m/s}^2}$$
$$t = 54 \text{ s}$$

The object has accelerated to 27 m/s in 54 seconds.

GRAVITY

The most common source of constant acceleration is due to the pull of the Earth's gravity. The acceleration of gravity near the surface of the earth is a constant, having a value of 32 ft/s^2 or 9.8 m/s^2. This acceleration is independent of the mass and shape of an object, which indicates that in a vacuum, where there is no air resistance, a feather will fall just as rapidly as a silver dollar. In the presence of air, however, a feather, a leaf, or a sheet of paper will flutter and fall more slowly because its shape and air resistance produce drag forces. The falling motion of heavier objects, such as stones and balls, is not affected significantly by the presence of air.

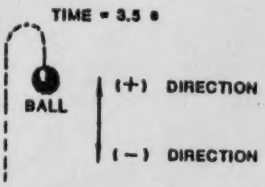
The value for acceleration of gravity decreases for an object very slowly, as it moves further in altitude from the earth's surface. There are also very small local variations at different points on the earth's surface, due to local variations in density of the earth below. The values given above are suitable in all cases dealt with in this module.

Problems in which velocity and acceleration are in different directions must be solved. This type of problem is solved in Example D, in which a ball is thrown vertically upward and is decelerated (or accelerated downwards) by the pull of gravity until it stops its upward motion and begins to fall, being accelerated downward.

EXAMPLE D: SPEED OF A BALL THROWN UPWARD

Given: A ball is thrown vertically upward with an initial speed of 18.6 m/s.

Find: Its speed after 3.5 seconds.



Solution:

$$v_2 = v_1 + at$$
$$= 18.6\ m/s + (-9.8\ m/s^2)(3.5s)$$
$$= 18.6\ m/s - 34.3\ m/s$$
$$v_2 = -15.7\ m/s$$

Since the final speed is negative, the ball is descending at 15.7 m/s after 3.5 seconds.
Notice that the upward direction was chosen as the positive direction; therefore the downward direction becomes the negative

direction. The initial speed was upward (positive), and the acceleration downward (negative). Since the final speed was negative, the ball travels downward. The direction chosen as positive was unimportant because the same sign convention was maintained throughout the solution.

SPECIAL CASES

If in Equation 1, 2 and 3 the initial speed is zero (that is, the body starts from rest), the equations can be shortened, in order that there will be only a single speed to consider. The equations become: $a = \frac{v}{t}$, $v = at$, and $t = \frac{v}{a}$. Memorization of these special cases as separate formulas is unnecessary, since $v_i = 0$ can be substituted into the original formulas to achieve the same result.

SUMMARY

Velocity, or speed, of an object at any time during its motion can be calculated if its initial speed and acceleration are known. Acceleration refers to the rate at which speed changes, and can be plotted graphically as the slope of velocity versus time. Velocity, however, can be plotted as the slope of distance versus time, or the distance traveled within a specific time.

DETERMINATION OF ACCELERATION

MATERIALS

Two pieces of aluminum angle (or angle iron) slightly over 2 m long.
One-inch diameter steel ball.
Two timers with at least 0.1 second accuracy.

PROCEDURE

In this experiement, a ball will be allowed to roll down an inclined track where it is accelerating, and then over a level track where it continues to move at constant velocity. (See Figure 2.) From data taken during the level portion of its motion, the velocity at the bottom of the incline can be determined. Once this data has been found, the constant acceleration of the ball during the inclined portion of its motion can be determined.

1. On each piece of aluminum angle, mark a distance of one meter from one end.

2. Position the two pieces of angle, as shown in Figure 2. One of the pieces is horizontal and the other rests inside it with its right end (B) one meter from the end of the horizontal piece (C). The other end of second length of angle is raised to form an angle of approximately twenty-five degrees with the horizontal.

3. Place steel ball at one meter mark on elevated angle (point A), as shown in Figure 2.

4. Perform in the following sequence:

   a. Release ball and start Timer One simultaneously.
   b. When ball reaches point B, stop

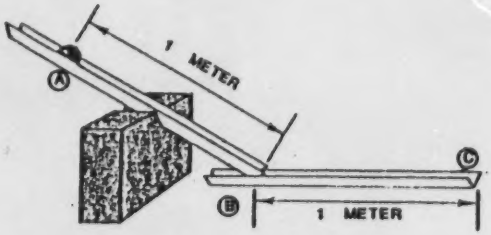


FIG. 2

Timer One and start Timer Two. (Suggestion: Use two people to start and stop timers.)

c. When the ball reaches point C, stop Timer Two.

5. Record times in Data Table 1.

6. Repeat experiment two more times and average times recorded.

7. Perform calculations necessary to complete Data Table 1.

8. Repeat experiment using 2 meter lengths for AB and BC, and increase the angle to about thirty degrees for AB. Record data on Data Table 2.

9. Complete Data Table 2.

10. How does this laboratory procedure confirm the principles in the Discussion?

DATA TABLE 1

DATA

Distance AB: d_1 = _____ m
Distance BC: d_2 = _____ m

		Trial 1	Trial 2	Trial 3	AVERAGE
Time from A to B	t_1				
Time from B to C	t_2				

Initial speed v_A = 0 m/s.

CALCULATIONS

To determine speed v_B:

$$v_B = v_C = \frac{d_2}{t_2} = _____ \text{ m/s}$$

To determine acceleration during descent:

$$a = \frac{v_B - v_A}{t_1} = _____ \text{ m/s}^2$$

DATA TABLE 2

DATA

Distance AB: d_1 = _____ m
Distance BC: d_2 = _____ m

		Trial 1	Trial 2	Trial 3	AVERAGE
Time from A to B	t_1				
Time from B to C	t_2				

Initial speed: $v_A = 0$ m/s

CALCULATIONS

To determine speed v_B:

$$v_B = v_C = \frac{d_2}{t_2} = _____ \text{ m/s}$$

To determine acceleration during descent:

$$a = \frac{v_B - v_A}{t_1} = _____ \text{ m/s}^2$$

SELF CHECK

1. An object is dropped from the top of a cliff. What is its speed after 5 seconds (neglect air resistance)?

2. A train accelerates from 40 km/hr to 50 km/hr in 30 seconds. What is its acceleration?

3. With a deceleration of 0.8 m/s^2, how much time is required to stop an object traveling at 16 m/s?

4. A constant acceleration is applied to an object starting from rest. After 8 seconds its speed is 12.6 m/s. How much longer must the same acceleration be applied to increase its speed to 28 m/s?

5. An object is thrown vertically upward with an initial velocity of 6.2 m/s. What is its speed after 1.2 seconds?

6. An object initially traveling to the right at 2.3 m/s was observed 4.8 seconds later traveling to the left at a speed of 12.8 m/s. What is the acceleration (magnitude and direction), assuming it has constant value? (Suggestion: Establish a sign convention for left and right directions.)

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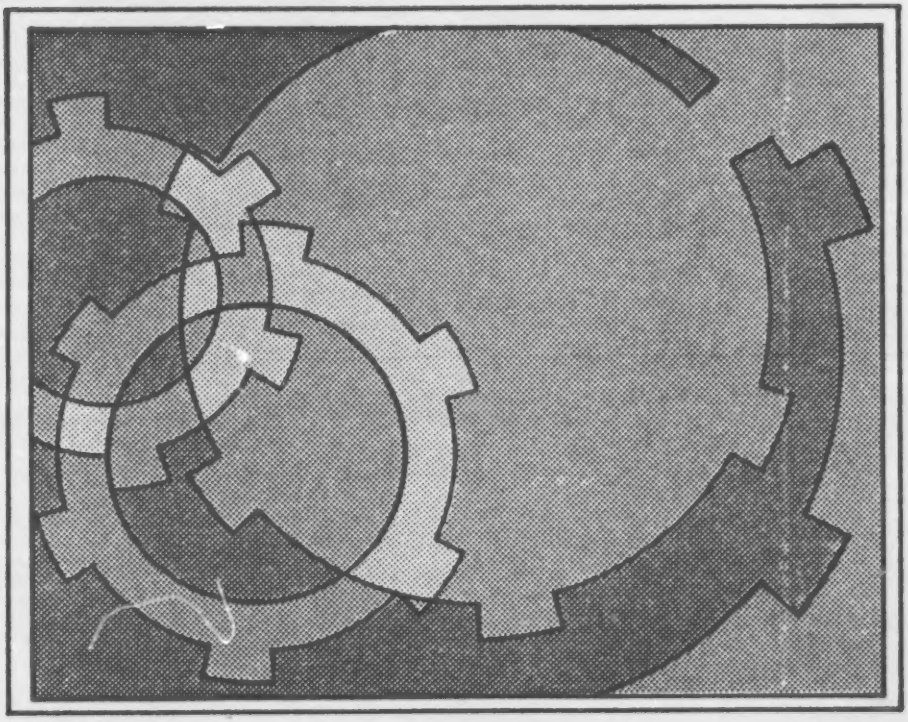
UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 3

DISPLACEMENT AT CONSTANT ACCELERATION



MECHANICAL APPLICATION OF RATE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

Everything in the physical world is in motion. In most cases, the velocity, or speed, of a moving object changes as motion continues. This change in velocity is called "accelerated motion." The simplest kind of acceleration is motion in a straight line, in which speed changes at a constant rate. This special kind of motion is called "uniformly-accelerated motion," or motion with constant acceleration. Displacement is the distance through which a body moves.

Acceleration is a "second-order" rate because it is the rate of a rate. More exactly, velocity is the rate at which displacement changes, and acceleration is the rate at which velocity changes. Hence, acceleration is a ratelike quantity of a more complex type than those explained in Concept Module 3-0, "Rate."

In this module, the student will learn that, given any three of the five motion parameters (displacement, initial velocity, final velocity, acceleration and time), the remaining two can be found. In the laboratory, the student will measure initial velocity, distance traveled and time traveled by an object to determine its acceleration.

Explanatory note: "Velocity" is a vector quantity whose magnitude is "speed." Although this module is confined to motion along a straight line, motion may be forward or backward. In certain instances, close attention must be given to this simple choice of directions, which is indicated by (+) and (-) signs attached to the speed. For this reason, the term "velocity" is preferable in most of the following discussion. At other times, the term "speed" will be employed.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate," and Application Module 3M2, "Velocity at Constant Acceleration."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given three of the following, determine the remaining two:

  Initial velocity, v_1.
  Final velocity, v_2.
  Constant acceleration, a.
  Elapsed time, t.
  Displacement, s.

- Define average velocity and displacement.

- Given the appropriate equipment, determine acceleration by measuring initial velocity and time required to travel a specific distance.

DISCUSSION

The definition of average velocity is the net displacement during a time interval divided by the length of that interval. This definition is expressed in Equation 1.

$$\bar{v} = \frac{s}{t} \qquad (1)$$

where

$\bar{v}$ = average velocity; bar over v indicates "average value."
s = net displacement, distance from starting point to final point.
t = time elapsed.

In the case of constant acceleration, average velocity also can be defined as the numerical average of the initial and final velocities, as shown in Equation 2.

$$\bar{v} = \frac{v_1 + v_2}{2} \qquad (2)$$

where

v_1 = initial velocity.
v_2 = final velocity.

Equations 1 and 2 are used to obtain Equation 3, which solves for net displacements at constant acceleration when v_1, v_2 and t are known.

$$s = \frac{(v_1 + v_2)\ t}{2} \qquad (3)$$

The final velocity of an object moving at constant acceleration is given by Equation 4.

$$v_2 = v_2 + at \qquad (4)$$

where

a = acceleration.

Equation 4 is substituted into Equation 3 as follows:

$$s = \frac{(v_1 + v_2)\ t}{2}$$

$$= \frac{[v_1 + (v_1 + at)]\ t}{2}$$

$$s = v_1 t + \frac{1}{2} at^2$$

This substitution yields the formula for displacement at a constant acceleration as given by Equation 5.

$$s = v_1 t + \frac{1}{2} at^2 \qquad (5)$$

Example A shows how Equation 5 can be used to solve a problem.

EXAMPLE A: DISPLACEMENT AT A CONSTANT ACCELERATION

Given: An object with an initial speed of 2 meters per second and an acceleration of 1 meter per second per second.

Find: How far it will travel in 5 seconds.

Solution:

$$s = v_1 t + \frac{1}{2} at^2$$

$$= (2 \tfrac{m}{s})(5\ s) + \frac{1}{2}(1 \tfrac{m}{s^2})(5\ s)^2$$

$$= 10\ m + \tfrac{1}{2}(25\ m)$$

$$s = 22.5\ m$$

The object will travel 22.5 meters in 5 seconds.

There are two terms in Equation 5. The first term, $v_1 t$, is the displacement during time t that would occur if the initial speed v_1 remained constant. The second term, $1/2\ at^2$, is the additional displacement due to speed gained through a constant acceleration a. Both of these terms are shown as dashed lines in Figure 1. Displacement at any time t is the sum of these two terms at that time. The graph of the $v_1 t$ term is a straight line, and that of the $1/2\ at^2$ term is curved (parabolic). The uppermost curve representing the sum, therefore, is also curved (again, parabolic).

In Figure 2, sample calculations are shown for the terms in Equation 5. Each term is calculated at the end of each second over a 5-second span. The body moves with an initial velocity of 2 m/s and a constant acceleration of 1 m/s^2.

Equation 4 solved for t gives $t = v_2 - v_1/a$. Substituting this into Equation 3 yields the following equation.

$$s = \frac{(v_2 + v_1)\ t}{2}$$

$$s = \frac{(v_2 + v_1)}{2} \quad \frac{(v_2 - v_1)}{a} \cdot t$$

$$s = \frac{v_2^2 - v_1^2}{2a}$$

$$2as = v_2^2 - v_1^2$$

This useful equation often is expressed as shown in Equation 6 for final velocity.

$$v_2^2 = v_1^2 + 2as. \qquad (6)$$

Example B shows the use of Equation 6 in solving a problem.

Example C shows the use of Equations 5 and 6 to solve a problem.

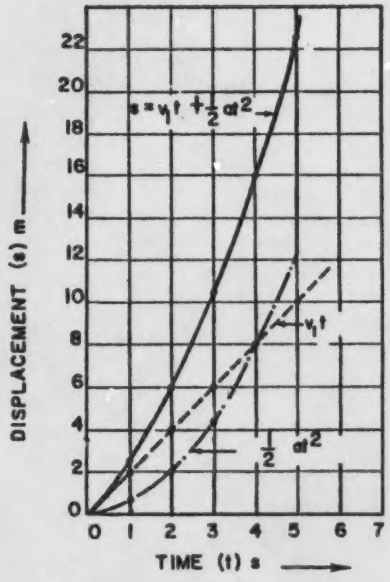


FIG. 1

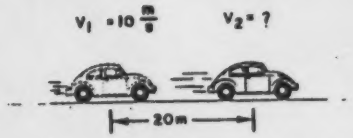
$v_1 = 2$ m/s,
$a = 1$ m/s^2.

t	$v_1 t$	$\frac{1}{2} at^2$	s
0	0	0	0
1	2	0.5	2.5
2	4	2	6
3	6	4.5	10.5
4	8	8	16
5	10	12.5	22.5

FIG. 2

EXAMPLE B: FINAL SPEED OF AUTOMOBILE

Given: An automobile with an initial speed of 10 m/s experiences an acceleration of 1.1 m/s² for a distance of 20 m.



Find: its final speed.

Solution:

$$v_2^2 = v_1^2 + 2as$$
$$= (10 \text{ m/s})^2 + 2\,(1.1 \text{ m/s}^2)(20 \text{ m})$$
$$= 100 \text{ m}^2/\text{s}^2 + 44 \text{ m}^2/\text{s}^2$$
$$= 144 \text{ m}^2/\text{s}^2$$
$$= \sqrt{144 \text{ m}^2/\text{s}^2}$$
$$v_2 = 12 \text{ m/s}$$

The car reaches a final speed of 12 m/s.

EXAMPLE C: ACCELERATION AND SPEED OF AN AUTOMOBILE

Given: An automobile undergoes a constant acceleration from an initial speed of 25 m/s for a period of 10 seconds and covers a distance of 500 m.

Find: (a) Acceleration of the car.
(b) Final speed.

Solution: (a) $s = v_1 t + \frac{1}{2} a t^2$; Solve for a.

$$a = \frac{25}{t^2} - \frac{2\,v_1}{t}$$
$$= \frac{2\,(500 \text{ m})}{(10 \text{ s})^2} - \frac{2\,(25 \text{ m/s})}{10 \text{ s}}$$
$$= \frac{1000 \text{ m}}{100 \text{ s}^2} - \frac{50 \text{ m/s}}{10 \text{ s}}$$
$$= 10 \text{ m/s}^2 - 5 \text{ m/s}^2$$
$$a = 5 \text{ m/s}^2$$

(b) $v_2^2 = v_1^2 + 2as$
$= (25 \text{ m/s})^2 + 2\,(5 \text{ m/s}^2)(500 \text{ m})$
$= 625 \text{ m}^2/\text{s}^2 + 5000 \text{ m}^2/\text{s}^2$
$v_2^2 = 5625 \text{ m}^2/\text{s}^2$

$$v_2 = \sqrt{5625\ m^2/s^2}$$
$$v_2 = 75\ m/s$$

The automobile had a constant acceleration of 5 m/s^2 and attained a final speed of 75 m/s.

Example D shows two approaches to solving a problem involving the acceleration of gravity.

EXAMPLE D: TIME TO REACH MAXIMUM HEIGHT

Given: A ball thrown vertically upward reaches a maximum height of 21 m.

Find: How long it takes to reach that height.

Solution: (a) Known: $s = 21\ m$
$v_2 = 0\ m/s$
$a = -9.8\ m/s^2$ (Upward was chosen as positive direction.)

$$v_2^2 = v_1^2 + 2as$$
$$v_1^2 = v_2^2 - 2as$$
$$= (0\ m/s)^2 - 2(-9.8\ m/s^2)(21\ m)$$
$$v_1^2 = 411.6\ m^2/s^2$$
$$v_1 = 20.29\ m/s$$

Ball is thrown upward at an initial velocity of 20.29 m/s.

$$v_2 = v_1 + at$$
$$t = \frac{v_2 - v_1}{a}$$
$$= \frac{20.29\ m/s - 0\ m/s}{9.8\ m/s^2}$$
$$t = 2.07\ s$$

It takes 2.07 seconds to reach a height of 21 meters.

(b) Assume that the ball requires the same time to travel the 21 meters in a downward direction as it does going upward. Therefore, starting at the top, $v_1 = 0\ m/s$.

$$s = v_i t + \frac{1}{2} a t^2$$

$$s = \frac{1}{2} a t^2$$

$$\frac{2s}{a} = t^2$$

$$t = \sqrt{\frac{2s}{a}}$$

$$= \sqrt{\frac{2\ (21\ m)}{9.8\ m/s^2}}$$

$$= \sqrt{\frac{42}{9.8}\ s^2}$$

$$= \sqrt{4.29\ s^2}$$

$$t = 2.07\ s$$

In this case, the downward direction is chosen as positive (+), so $a = +9.8\ m/s^2$, the same as in solution a.

SUMMARY

Displacement, the distance through which a body moves, is simple to calculate when the velocity of the moving body changes at a constant rate (at a constant acceleration). Displacement often must be determined for moving objects at a given speed and time. If any three of the following motion parameters are known, the remaining two can be found: displacement, initial velocity, final velocity, acceleration and time.

DETERMINING ACCELERATION AND DISPLACEMENT

MATERIALS

Two pieces of aluminum angle slightly over 2 m long or other suitable track on which a steel ball can roll.
One-inch diameter steel ball.
Two timers with an accuracy of one tenth second or better.

PROCEDURE

Acceleration of a ball rolling down an incline will be determined experimentally and then used to determine the distance the ball will roll down the incline in a given time.

1. On one piece of track, mark a distance of one meter from one end. On longer piece, mark a distance of 2 m from one end. Make another mark approximately 20 cm from 2-m mark.

2. Position two pieces of track as shown in Figure 3. Horizontal piece should be elevated so that slanted piece will make an angle of approximately 10 degress with horizontal.

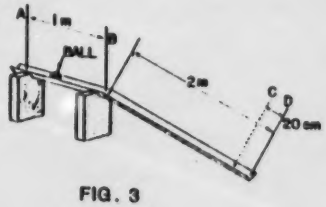


FIG. 3

3. Place ball at point B on end of slanted piece of track. Release ball; let it roll down incline. Measure and record in Data Table amount of time required for ball to travel 2 m. Repeat this three times; record average time in Data Table.

4. Calculate ball acceleration.

5. Place steel ball at left end of horizontal piece; roll it toward other end. (Be careful not to cause any acceleration after point A.)

6. Perform following steps as ball rolls:
   When ball passes point A, start timer one.
   When ball passes point B, stop timer one; start timer two.
   When ball passes point C, stop timer two.

7. Record results in Data Table.

8. Perform all calculations necessary to complete that section of Data Table.

9. Measure distance BC; record in Data Table.

10. How does this laboratory procedure confirm the explanation of displacement in the Discussion?

DATA TABLE

Acceleration Calculation

Distance traveled: $s = 2m$
Initial velocity: $v_1 = 0$ m/s

Time required: $t =$ _____ s

Acceleration: $s = v_1 t + \frac{1}{2} at^2$ $(v_1 t = 0)$

$a = \frac{2s}{t^2}$

$= 2 \times 2\ m \div (_____\ s)^2$

$=$ _____ m/s^2

Displacement Calculation

Distance AB: $d_1 =$ _____ m

Time for A to B: $t_1 =$ _____ s

Time for B to C: $t_2 =$ _____ s

Velocity at B: $v_1 = \frac{d_1}{t_1}$

$=$ _____ m $\div$ _____ s $=$ _____ m/s

Distance traveled (BC): $s = v_1 t_2 + \frac{1}{2} at_2^2$

$=$ _____ m/s $\times$ _____ s $+ \frac{1}{2} \times$ _____ $m/s^2 \times (_____ s)^2$

$=$ _____ m

Distance BC (measured): $d_2 =$ _____ m

SELF CHECK

1. A ball is dropped from a window. How far will it fall in 3 seconds?

2. A rock is dropped over a cliff. What is its velocity after it has fallen 5.7 m?

3. An arrow was shot vertically upward. As it fell back to the ground, it had a velocity of 8.6 m/s at a height of 5.1 m.

   What was the initial velocity of the arrow?
   How high did it go?
   How much time was required for the ball to fall from its maximum height to the ground?

4. A car traveling at 30 m/s stops with a constant deceleration applied for 10 seconds. How far did it travel in that 10 seconds?

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Harris, Norman C. and Hemmerling, Edwin M. Introductory Applied Physics. 3rd ed. New York: McGraw-Hill Book Company, 1972. See especially Chapter 3.

Sears, F.W. and Zemansky, M.W. Modern College Physics, Part I: Mechanics and Thermodynamics. Reading, MA: Addison-Wesley Publishing Company, 1962.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 4

ACCELERATION OF A MASS BY A CONSTANT FORCE

MECHANICAL APPLICATION OF RATE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Whenever a net force acts upon an object, the object accelerates. The magnitude of the acceleration depends upon the mass of the object and the size of the acting force. If two unequal forces act alternately upon the same object, the larger force produces the greater acceleration. In addition, if the same force acts alternately upon objects of different mass, the object with the smaller mass acquires the greater acceleration.

This module describes how the parameters of force, mass, and acceleration relate to each other and to the remaining four motion parameters (displacement, initial velocity, final velocity, and time). In the laboratory, the student will measure the initial velocity, distance traveled, and time of travel of an object to determine its acceleration and, from this, its mass.

Many situations involve forces whose magnitudes change while they are acting on an object. The mathematical description of such changing forces is beyond the scope of this discussion; therefore, only constant forces and their resultant accelerations will be presented. In the laboratory, the student will construct an Atwood's Machine and will use it to determine the mass of an object.

PREREQUISITES

Student should have completed Concept Module 4-0, "Rate"; and Application Modules 3M2," Velocity at a Constant Acceleration," and 3M3, "Displacement at a Constant Acceleration."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given any two of the following quantities, calculate the third:

  Force acting upon an object.

Mass of object.
Acceleration of object.

- Given four of the following quantities, determine the remaining two:

  Initial velocity, v_1.
  Final velocity, v_2.
  Constant force, F.
  Mass, m.
  Elapsed time, t.
  Displacement, s.

- Given the appropriate equipment, construct an Atwood's Machine, and use it to determine the mass of an object.

ACCELERATION DUE TO A SINGLE FORCE

The acceleration of an object always is directly proportional to the net force acting upon the object and inversely proportional to the object's mass. The formula for calculating the acceleration of an object acted upon by a force is given below:

$$a = \frac{F}{m} \qquad (1)$$

where

a = acceleration.
F = force acting upon object.
m = mass of object.

Example A illustrates how Equation 1 is used to solve a problem.

EXAMPLE A: ACCELERATION OF A CART

Given: An empty cart of mass 20 kilograms and a cart of mass 10 kilograms are pushed with a constant force of 50 Newtons.

Find: The acceleration of each cart.

Solution: For the larger cart acceleration is –

$$a = \frac{F}{m}$$

$$= \frac{50\ N}{20\ kg}$$

$$= \frac{50\ \frac{kg \times m}{s^2}}{20\ kg}$$

$$a = 2.5\ m/s^2$$

For the smaller cart acceleration is –

$$a = \frac{F}{m}$$

$$= \frac{50\ N}{10\ kg}$$

$$= \frac{50\ \frac{kg \times m}{s^2}}{10\ kg}$$

$$a = 5\ m/s^2$$

Example A illustrates how objects of small mass, when acted upon by a force, achieve greater accelerations than more massive objects acted upon by the same force. The mass of the small cart clearly is one-half that of the larger, and it accelerates at twice the rate of the large cart. Finally, the force and the mass must be expressed in the same system of units. In the SI system, mass is expressed in kilograms; force, in Newtons; and acceleration, in meters per second per second (m/s^2).

Example B illustrates the accelerations of equal masses by forces of differing magnitudes.

EXAMPLE B: ACCELERATION OF OBJECTS OF EQUAL MASS BY DIFFERENT FORCES

Given: A 1.6-kilogram object is acted upon by an 8-N force.
A different 1.6-kg object is acted upon by a 4-N force.

Find: The acceleration of each object.

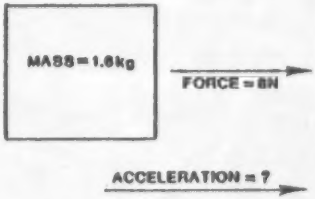
Solution: For the first object –

$$a = \frac{F}{m}$$

$$= \frac{8\ N}{1.6\ kg}$$

$$= \frac{8\ \frac{kg \times m}{s^2}}{1.6\ kg}$$

$$a = 5\ m/s^2$$



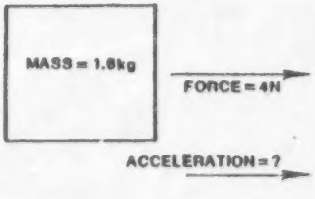
For the second object –

$$a = \frac{F}{m}$$

$$= \frac{4\ N}{1.6\ kg}$$

$$= \frac{4\ \frac{kg \times m}{s^2}}{1.6\ kg}$$

$$a = 2.5\ m/s^2$$



Examples A and B demonstrate the relation between force, mass, and acceleration. Table I shows the resultant accelerations when three different forces act upon each of three different masses. When the accelerations are plotted as the y-coordinate and the forces are plotted as the x-coordinate, several outstanding features are apparent immediately. Primarily, all of the points corresponding to a given mass lie on the same straight line. In addition, the lines corresponding to each different mass are unique.

TABLE 1: RELATIONSHIPS OF MASS, FORCE AND ACCELERATION

Mass of object In kilograms	Force acting in Newtons	Acceleration in m/s^2
1	6	6
1	12	12
1	18	18
2	6	3
2	12	6
2	18	9
3	6	2
3	12	4
3	18	6

Finally, the slope, or steepness of the lines corresponding to each mass increases with decreasing mass. The mass of a particular object then can be determined from a knowledge of its acceleration and the force acting upon it; for instance, if an object is acted upon by a 15-Newton force and accelerates at a rate of 7.5 m/s^2, its force and acceleration can be plotted on Figure 1 at Point A. Since Point A falls directly on the line that corresponds to a mass of 2 kilograms, the mass of the object must be 2 kilograms.

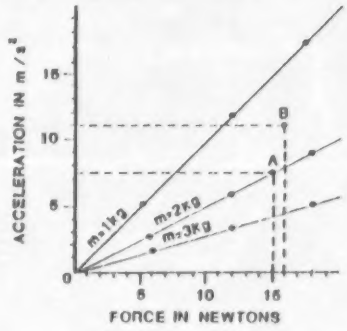


FIG. 1

A second object accelerates at the rate of 11 m/s^2 while being acted upon by a force of 16 Newtons. This object's force and acceleration are plotted as Point B on Figure 1. Because Point B does not fall directly on any of the mass lines, an exact determination of the object's mass cannot be made from the graph. It is possible, however, to estimate the mass of

the object as being between one and two kilograms.

A more precise value of the mass of the object can be determined by plotting Point B on a graph having many more mass lines. There is, however, a better and simpler way of obtaining a precise value for the mass. This method involves solving Equation 1 for mass to obtain Equation 2:

$$a = \frac{F}{m}$$

$$ma = F \text{ (multiplying both sides by m)}$$

$$m = \frac{F}{a} \text{ (dividing both sides by a)} \qquad (2)$$

Example C illustrates the use of this equation in determining the mass of an object.

EXAMPLE C: MASS OF AN OBJECT TRAVELING WITH CONSTANT ACCELERATION

Given: A force of 16 Newtons acts upon an object accelerating at the rate of 10 m/s^2

Find: Its mass.

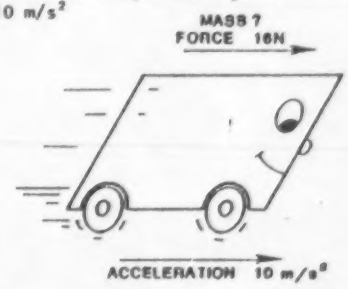
Solution:

$$m = \frac{F}{a}$$

$$= \frac{16 \text{ N}}{10 \text{ m/s}^2}$$

$$= \frac{16 \frac{\text{kg} \times \text{m}}{\text{s}^2}}{10 \frac{\text{m}}{\text{s}^2}}$$

$$m = 1.6 \text{ kg}$$



Example C demonstrates how the mass of an object can be found directly, without graphs, from the object's acceleration and the force acting upon it. The mass of the object represented by Point B in Figure 1 now can be calculated:

$$m = \frac{F}{a} = \frac{16 \text{ N}}{11 \text{ m/s}^2} = 1.45 \text{ kg}$$

The force necessary to cause an object to accelerate at a given rate sometimes must be determined. This formula is obtainable from Equation 1 when rearranged in the form of Equation 3.

$$a = F/m$$
$$ma = F \text{ (multiply both sides by m)}$$

or

$$F = ma \qquad (3)$$

Equation 3 is the most commonly-used form of the three equivalent formulas. Its use is shown in Example D.

EXAMPLE D FORCE AT CONSTANT ACCELERATION

Given: A 2-kilogram object accelerates at the rate of 10 m/s^2.

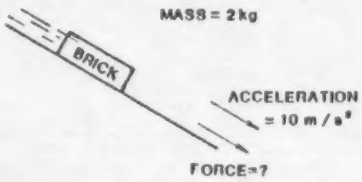
Find: The force acting upon it.

Solution: $F = ma$

$= 2 \text{ kg} \times 10 \text{ m/s}^2$

$= 20 \text{ kgm/s}^2$

$F = 20 \text{ N}$



PROBLEMS INVOLVING MORE THAN ONE FORCE

Up to this point, force and acceleration have been dealt with as though they were scalars. For cases in which only one force acts, this treatment is adequate, since the direction of the acceleration must be the same as the direction of the force. If two forces are acting — in opposite directions, for instance — the vector nature of the forces and their accelerations must be taken into account.

Figure 2 illustrates a heavy box of mass m being pulled in opposite directions by two men. The first man exerts force $\vec{F}_1$ on the box, and the second man exerts force $\vec{F}_2$. If only the first man were pulling, Equation 3 could be written $\vec{F}_1 = m\vec{a}_1$. In this case, the acceleration of the box is given by Equation 1 as follows:

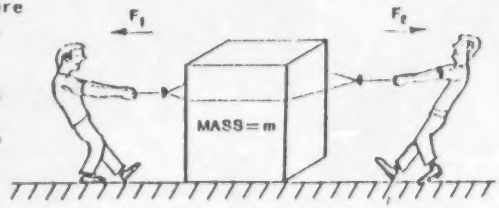


FIG. 2

$$|\vec{a}_1| = \frac{|\vec{F}_1|}{m}$$

or, $$\vec{a}_1 = \frac{|\vec{F}_1|}{m} \quad \text{to the left} \qquad (4)$$

If only the second man pulls, the following is the result:

$$\vec{F} = m\vec{a}_2 \qquad (5)$$

so, $$|\vec{a}_2| = \frac{|\vec{F}_2|}{m}$$

and, $$\vec{a}_2 = \frac{|\vec{F}_2|}{m} \quad \text{to the right}$$

When both men pull simultaneously, the total force acting upon the box is the vector sum of $\vec{F}_1$ and $\vec{F}_2$. Equation 6 is formed, therefore, when Equations 4 and 5 are added.

$$\begin{array}{l} \vec{F}_1 = m\vec{a}_1 \\ \vec{F}_2 = m\vec{a}_2 \\ \hline \vec{F}_1 + \vec{F}_2 = m\vec{a}_1 + m\vec{a}_2 = m(\vec{a}_1 + \vec{a}_2) \end{array} \qquad (6)$$

Since $\vec{F}_1$, $\vec{F}_2$, $\vec{a}_1$, and $\vec{a}_2$ all act along the same line, quantities directed to the right ($\vec{F}_2$, $\vec{a}_2$) can be considered positive, and quantities directed to the left ($\vec{F}_1$, $\vec{a}_1$) considered negative. Equation 6 then becomes Equation 7.

$$F_2 - F_1 = m(a_2 - a_1) = ma \qquad (7)$$

The quantity $F_2 - F_1$ is called the net, or unbalanced, force. The quantity a is the net acceleration due to this force. Whenever two forces act in opposition, the net force is the difference in their magnitudes; and the object accelerates in the direction of the larger force.

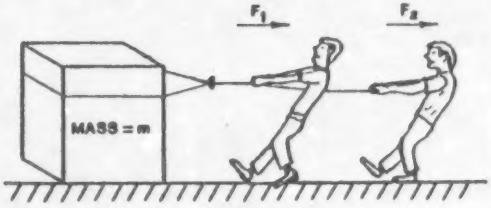


FIG. 3

Suppose both men pull in the same direction (Figure 3). Then Equation 6 becomes $F_1 + F_2 = ma$, where right again has been chosen as positive. In this case the unbalanced force is the sum of the magnitudes of the individual forces.

In many mechanical systems, a force must be exerted in order to keep

an object moving at a constant speed; for example, a cyclist must pedal to maintain a constant speed on level ground. In all such cases, there is at least one additional force acting in opposition to the applied force. This force often is caused by friction. If the force exerted by the cyclist is exactly equal to the opposing force of friction, the unbalanced force has a magnitude of zero, resulting in no acceleration (constant speed). Examples E and F demonstrate how the concept of unbalanced force can be used to solve a problem.

EXAMPLE E: ACCELERATION OF AN ELEVATOR

Given: A winch exerts an upward force of 2000 Newtons on a 200-kilogram elevator.

Find: Its acceleration.

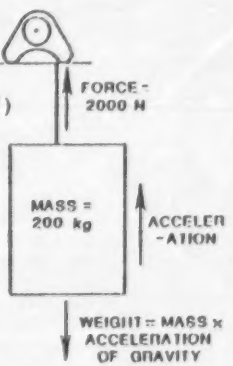
Solution: First find the unbalanced force on the elevator. Choose upward as positive. Then the force down on the elevator is equal to its weight:

$$W = mg$$

where
W = weight.
m = mass.
g = acceleration due to gravity.

Therefore,

$$W = mg$$
$$= (200\ kg)(-9.8\ m/s^2)$$
$$W = -1960\ N$$



The unbalanced force is then —

unbalanced force = upward force + downward force (vector sum)

$$F = F_{up} - W$$
$$= 2000\ N - 1960\ N$$
$$F = 40\ N$$

Now find the acceleration:

$$a = \frac{F}{m}$$

$$a = \frac{40\ N}{200\ kg}$$ (dividing both sides by 200 kg)

$$a = \frac{40 \frac{kg\ m}{s^2}}{200\ kg}$$

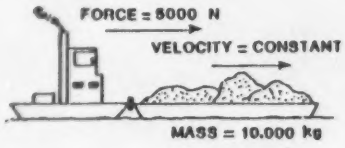
$$a = +0.2\ m/s^2$$

The acceleration of the elevator is 0.2 m/s^2.

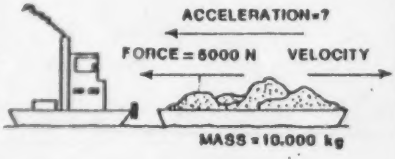
The positive sign indicates that the direction of the acceleration is upward.

EXAMPLE F: FORCES ON A BARGE

Given: A 10,000-kilogram barge is pushed by a tugboat with a force of 5000 Newtons. The barge travels through the water with constant speed.



Find:
(a) The force exerted against the barge by the water.
(b) The acceleration of the barge if the tugboat suddenly stops. F = ma.



Solution: (a) Notice that acceleration = 0, since the barge travels with constant speed. Thus,

$$\text{Net force} = 10{,}000\ kg \times 0\ m/s^2 = 0\ N$$

Let the force of the water on the barge be Fw. Choose the direction of motion of the barge as positive. Then,

$$5000\ N - Fw = 0$$
$$Fw = 5000\ N$$

The magnitude of the force of the water against the barge is 5000 N.

(b) When the tugboat stops, the only force acting is the force of the water on the barge. Choose the direction of the barge as positive.

$$F = ma$$
$$-5000\ N = (10000\ kg)(a)$$
$$a = \frac{-5000\ N}{10000\ kg}$$
$$= \frac{-5000\ \frac{kg \times m}{s^2}}{10000\ kg}$$
$$a = -0.5\ m/s^2$$

The magnitude of the acceleration of the barge is 0.5 m/s^2. The negative sign indicates that the barge is decelerating (slowing down).

PROBLEMS INVOLVING DISPLACEMENT, VELOCITY, AND TIME

There exists a whole class of different problems for which the force on an object must be calculated without a direct knowledge of the object's acceleration. In these cases, combinations of measurements of displacement, initial velocity, final velocity, and elapsed time are made instead of measurements of the acceleration. These problems are of tremendous practical importance because there are very few instruments that measure acceleration directly, although instruments for measuring displacement, velocity, and time are quite commonplace.

This dilemma easily is solved if one realizes that the acceleration of any object can be expressed as the net force acting upon the object divided by the object's mass: $a = F/m$.

All the equations in Application Modules 3M2 and 3M3 that relate acceleration to displacement, velocity, and time can be modified by replacing the acceleration a by the equivalent quantity F/m. These equations are listed in their original and modified forms in Table 2.

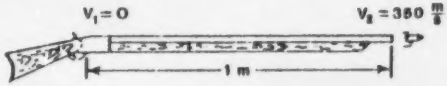
TABLE 2: EQUATIONS OF MOTION	
Motion Equation in terms of acceleration (a)	Motion Equation in terms of force (F) and mass (m)
$a = \frac{v_2 - v_1}{t}$	$\frac{F}{m} = \frac{v_2 - v_1}{t}$
$v_2 = v_1 + at$	$v_2 = v_1 + \frac{F}{m} t$
$t = \frac{v_2 - v_1}{a}$	$t = \frac{v_2 - v_1}{F/m} = \frac{m(v_2 - v_1)}{F}$
$s = v_1 t + \frac{1}{2} at^2$	$s = v_1 t + \frac{1}{2} \frac{F}{m} t^2$
$v_2^2 = v_1^2 + 2 ad$	$v_2^2 = v_1^2 + 2 \frac{F}{m} s$

Examples G and H indicate how these equations are utilized in solving problems.

EXAMPLE G: THE FORCE ON A BULLET

Given: A 50-gram bullet is fired from a rifle. The length of the barrel is 1 meter, and the bullet emerges with a speed of 350 m/s.

Find: The force on the bullet while in the barrel.



Solution: The bullet starts from rest; so $v_1 = 0$. Convert the mass of the bullet into SI units:

$$(50 \text{ g})\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = \frac{50}{1000} \text{ kg} = 0.05 \text{ kg}$$

Then

$$v_2^2 = v_1^2 + 2 \frac{F}{m} s$$

$$(350 \text{ m/s})^2 = 0^2 + 2 \left(\frac{F}{0.05 \text{ kg}}\right)(1 \text{ m})$$

$$\frac{(350\ m/s)^2}{2m} = \frac{F}{0.05\ kg} \quad \text{(dividing both sides by 2 m)}$$

$$F = \frac{(0.05\ kg)(350\ m/s^2)}{2m} \quad \text{(multiplying both sides by 0.05 kg and reversing sides)}$$

$$= \frac{(0.05\ kg)(122{,}500\ \frac{m^2}{s^2})}{2m}$$

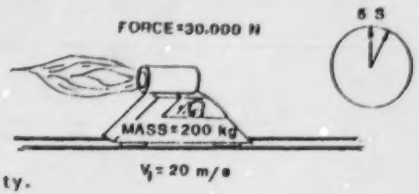
$$= (3062.5\ kg)(m/s^2)$$

$$F = 3062.5\ N$$

Notice that force is assumed to be constant.

EXAMPLE H: FINAL VELOCITY OF A ROCKET SLED

Given: A 200-kilogram rocket sled is acted upon by a 30,000-Newton force for 5 seconds. The initial velocity of the rocket sled is 20 m/s.



Find: Its final velocity.

Solution:

$$v_2 = v_1 + \frac{F}{m}t$$

$$v_2 = 20\ m/s + \frac{(30{,}000\ N)(5\ s)}{200\ kg}$$

$$= 20\ m/s + \frac{(30{,}000\ \frac{kg \times m}{s^2})(5\ s)}{200\ kg}$$

$$= 20\ m/s + 750\ m/s$$

$$v = 770\ m/s$$

The last topic to be considered in this module is a simple device called the "Atwood's Machine," which consists of two unequal masses tied to opposite ends of a string that passes over a pulley. When the masses are released, the heavy mass falls, and the light mass rises. The acceleration of the masses can be calculated from the principles presented in this module.

Consider an Atwood's machine constructed from a heavy mass m_1 and a light mass m_2 (see Figure 4). Because the masses are tied together, both must travel with the same acceleration a. Figure 5 depicts the forces acting upon each mass. As previously noted, the acceleration of any object is equal to the unbalanced force acting upon it divided by its mass. Then, for mass m_1 Equation 8 holds true.

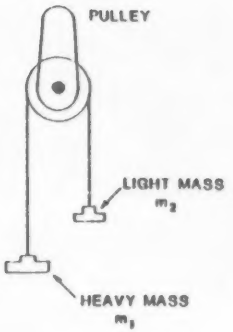


FIG. 4

$$m_1 g - T = m_1 a \qquad (8)$$

and for mass m_2, Equation 9 is used.

$$T - m_2 g = m_2 a \qquad (9)$$

Addition of Equations 8 and 9 gives the following:

$$m_1 g - T + T - m_2 g = m_1 a + m_2 a$$

$$m_1 g - m_2 g = m_1 a + m_2 a$$

Suppose that the mass of one of the objects (say m_2) is known and that the other (m_1) is not. The Atwood's Machine can be used as a crude scale to measure the mass m_1. When the above equation is solved for m_1, Equation 10 is produced.

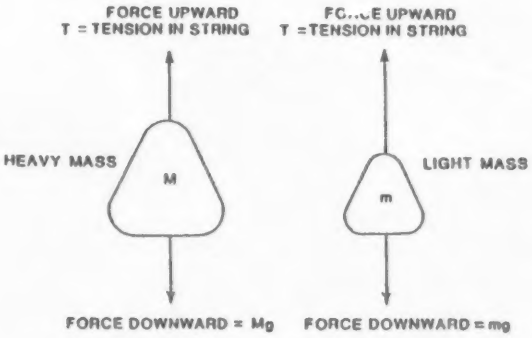


FIG. 5

$$m_1 = \frac{m_2(g + a)}{(g - a)} \qquad (10)$$

The acceleration can be determined from the measurements of the distance traveled d by one of the objects and the elapsed time. If the objects start from rest, the distance traveled is given by Equation 11.

$$d = \frac{1}{2} a t^2 \qquad (11)$$

$$a = \frac{2s}{t^2}$$ (multiplying both sides by 2, dividing by t^2 and reversing sides)

Acceleration is calculated from values of displacement and time derived from Equation 11. This value of acceleration then is substituted into Equation 10 to find the value of the unknown mass.

Several comments on the use of Atwood's Machine to measure mass are in order. First, the above analysis and Equation 10 can be used only if the unknown mass is greater than the known mass.

A second important point involves the fact that the mass of an object determined from measurements of the acceleration of an Atwood's Machine generally is lower than the true mass (as measured on a balance). This problem arises mainly due to friction in the pulley, which results in the acceleration of the Atwood's Machine being reduced. Depending on the friction inherent in the pulley used, the measured values for the true mass can be as much as 10% smaller than the "true" value.

DETERMINING THE MASS OF AN OBJECT BY USE OF AN ATWOOD'S MACHINE

MATERIALS

Two objects — one of known mass and one of unknown mass.
One light, relatively-frictionless pulley.
About 2 meters of light string.
One timer with accuracy of 0.1 s or better.
One balance with accuracy of 1 g or better.
One 2-meter stick.
Set of standard hooked masses.

PROCEDURE

In this experiment, an Atwood's Machine will be constructed from an object of known mass and an object of unknown mass. The machine will be released from rest; and, from data taken while it is accelerating, its acceleration will be determined. From this data the mass of the unknown object can be found.

1. After obtaining object of unknown mass m_1 choose known mass m_2 from a set of standard hooked masses. Remember $m_1 > m_2$. (A good rule of thumb is to keep both masses below 100 g.) Record the value of m_2 (known mass) in Data Table.

2. Hang pulley approximately 2 meters above floor. Tie objects to opposite ends of string, and pass string over pulley (Figure 6).

3. With heavy object resting on floor directly below pulley, measure distance AB from floor to bottom of light mass. Record this distance in Data Table.

4. Pull small mass down and hold it firmly against floor directly below pulley.

5. Release small mass; simultaneously start timer.

6. When large mass strikes floor, stop timer. Record time.

7. Repeat experiment two more times; average the times recorded.

8. Weight unknown mass on the balance. Record its true mass.

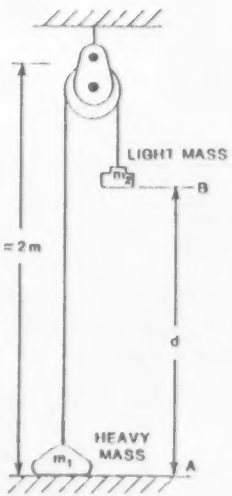


FIG. 6

9. Perform calculations necessary to complete Data Table.

10. If the value of the calculated mass differs from that of the true mass, explain how the equipment may have caused this difference.

DATA TABLE

Known mass: m = _____ g

Distance AB: d = _____ m

	Trial 1	Trial 2	Trial 3	Average
Time from AB to B: t =				

True mass of unknown object: _____

m_1 = _____ g

Calculations:

To determine acceleration, a: _____

$a = 2d/t^2$ = _____ m/s^2

To determine mass of unknown object, m_1.

$$m = \frac{m(9.8\ m/s^2 + a)}{(9.8\ m/s^2 - a)} = _____\ g$$

SELF CHECK

1. A man exerts a force of 100 N on a 10-kg wagon. Friction exerts a force of 20 N in the opposite direction. What is the acceleration?

2. An object accelerates at the rate of 5 m/s^2 when acted upon by a 50-N force. What is the object's mass?

3. A boy pushes a 10-kg sled with a constant force of 25 N for 2 s. What is the final velocity of the sled?

4. A 4000-kg truck initially traveling at 10 m/s brakes uniformly to a stop in a distance of 20 m. What braking force was applied to stop the truck?

5. Each of two horses exerts a force of 1000 N in the same direction on a 200-kg wagon. Assuming no friction, what is the unbalanced force on the wagon?

6. A winch exerts an upward force of 10000 N on an 1100-kg elevator. What is the magnitude and direction of the acceleration of the elevator?

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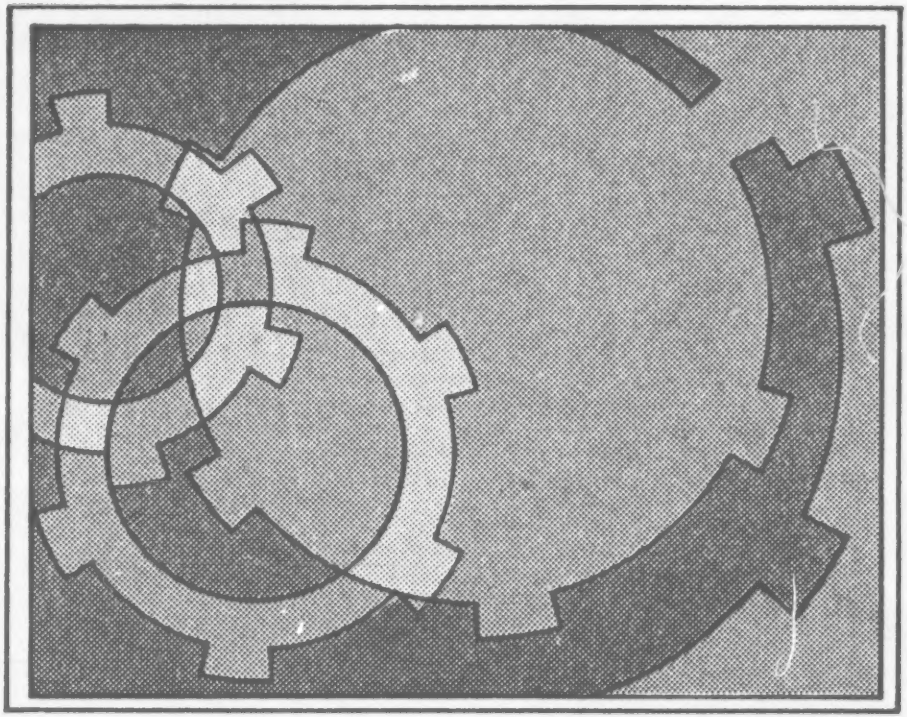
UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 5

ROTATIONAL MOTION



MECHANICAL APPLICATION OF RATE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

The study of motion is incomplete without consideration of rotational motion, since the most general type of motion is a combination of translation and rotation. Everything on the earth, whether moving of standing still with respect to the earth, undergoes rotation because the earth rotates on its axis and revolves about the sun. Since the discovery of the wheel, man has been concerned with the problem of rotational motion from a practical standpoint.

In this module, purely-rotational motion will be analyzed. The equations of motion governing rotation related angular displacement, angular velocity, and angular acceleration and are analogous to those which describe rectilinear, or translational, motion. In the laboratory, the student will use the equations and principles of rotation to determine the angular velocity and angular acceleration of a phonograph.

PREREQUISITES

The student should have completed Concept Module 3-0, "Rate," and Application Module 3M3, "Displacement at Constant Acceleration."

OBJECTIVES

Upon completion of this module, the student should be able to:

- State and define the angular analogue of each of the following:

  Initial velocity, v_1.
  Final velocity, v_2.
  Constant acceleration, a.
  Elapsed time, t.
  Displacement, s.
  $v_2 = v_1 + at$.
  $s = v_1 t + \frac{1}{2} at^2$.
  $v_2^2 = v_1^2 + 2as$.

- Given linear variables s and v_1 and the radius of rotation r, calculate angular counterparts θ and ω.

- Given three of the following, determine the remaining two:

  Initial angular velocity, ω_1.
  Final angular velocity, ω_2.
  Constant angular acceleration, α.
  Elapsed time, t.
  Angular displacement, θ.

- Given the appropriate equipment, determine the angular velocity and acceleration of a phonograph.

Figure 1 represents a rigid body, of arbitrary shape, that rotates about a fixed axis perpendicular to the plane of the figure at point Q. The position of the rotating body at any instant of time t is specified by the angle θ made by its radius vector QR with respect to a reference axis QX in the diagram. The coordinates r and θ are analogous to the rectangular coordinates x and y. The angle θ is defined conveniently in radians by Equation 1.

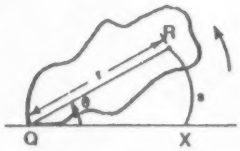


FIG. 1

$$\theta = \frac{s}{r} \qquad (1)$$

where

- θ = any arbitrary angle subtended by an arc of length s.
- s = arc length on circumference of circle of radius r.
- r = radius.

As shown in Figure 2, the radian is a unit that has no physical dimensions, since it is the ratio of two lengths. For a complete revolution $s = 2\pi r$, and $\theta = 2\pi$; therefore, there are 2π radians in a circle, or 2π radians = 360°.

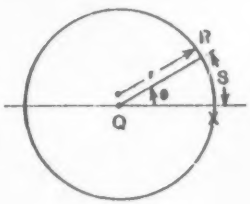


FIG. 2

If the rotating body requires time t to move from point X to point R, the arc length s is the linear displacement in the time interval t. Linear velocity is stated by Equation 2.

$$v = \frac{s}{t} \qquad (2)$$

where

- v = linear velocity.
- s = arc length — in this case, linear displacement.
- t = elapsed time for displacement s.

Equation 1 relates the linear displacement to the angle θ, which is the angle through which the body rotates in the time interval t, or the angular displacement:

$$\theta = \frac{s}{r} \qquad (1)$$

$$s = r\theta \qquad \text{(Both sides multiplied by r)}$$

$$v = \frac{r\theta}{t} \qquad \text{(rθ substituted for s in Equation 2)}$$

The quantity $\frac{\theta}{t}$ is the angular displacement θ divided by the elapsed time t, and is defined as the "angular velocity." The relationship between angular velocity and linear velocity is shown by Equation 3, where ω is substituted for θ/t.

$$v = r\omega \qquad (3)$$

where

v = linear velocity.
ω = angular velocity.

In this equation, angular velocity must be expressed in radians divided by time units, usually seconds. A more common unit for angular velocity is revolutions per minute, abbreviated r/min or rpm. The conversion from rpm to rad/s is —

$$1 \text{ r/min} = 2\pi \text{ rad/60 s},$$

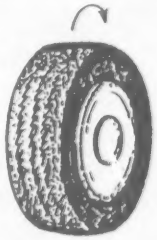
since one revolution is one trip around a circle or 2π radians, and since one minute is 60 seconds. Example A illustrates the use of Equations 1 and 3 in analyzing the rotation of automobile tires.

EXAMPLE A: ROTATION OF AUTOMOBILE TIRES

Given: An automobile with tires of 0.35 m radius moves at a speed of 18.0 m/s and travels a distance of 1.6 km. The rim speed of the tire must equal the translational speed of the car — in this case, 18.0 m/s.

Find: (a) The angular velocity of the tires in rad/s and in r/min.

(b) The number of revolutions made by the tires.



Solution: (a) The angular velocity in rad/s is found from Equation 3:

$$\omega = \frac{v}{r} = \frac{\text{rim speed of tire}}{\text{radius of tire}}$$

$$= (18.0 \text{ m/s})/(0.35 \text{ m})$$

$$\omega = 51.4 \text{ rad/s}$$

This result is converted to rpm (r/min):

$$\omega = (51.4)(60/2\pi)$$

$$\omega = 490.8 \text{ r/min}$$

(b) The angular displacement θ is found from Equation 1:

$$\theta = \frac{s}{r}$$

$$= (1.6 \times 10^3 \text{m})(0.35 \text{ m})$$

$$\theta = 4.57 \times 10^3 \text{ rad}$$

There are 2π radians in one revolution. The number of revolutions made in 4.57×10^3 radians is then $4.57 \times 10^3/2\pi$, or about 727.

ROTATION AT CONSTANT ANGULAR ACCELERATION

If the angular velocity is not constant, but changes from initial value to final value during a time interval, the rotating body undergoes angular acceleration. Angular acceleration is defined as the change in the angular velocity divided by the elapsed time, as indicated by Equation 4.

$$\alpha = \frac{(\omega_2 - \omega_1)}{t} \qquad (4)$$

where

α = angular acceleration.
ω_1 = initial angular velocity.
ω_2 = final angular velocity after time interval t.

Equation 4 can be rearranged to solve for final angular velocity as shown by the following process that results in Equation 5:

$$\alpha t = \omega_2 - \omega_1 \quad \text{(Both sides multiplied by t)} \qquad (4)$$

$$\omega_1 + \alpha t = \omega_2 - \omega_1 + \omega_1 \quad (\omega_1 \text{ added to both sides})$$

$$\omega_2 = \omega_1 + \alpha t \qquad (5)$$

Equation 5 is analogous to the relation for the final velocity for [illegible] translation at constant acceleration: $v_2 = v_1 + at$.

A rotating body undergoes constant angular acceleration; for example, if the direction of rotation is changed. If the direction of rotation is initially counterclockwise, and the magnitude is initially ω_1, and the final angular velocity is ω_1 clockwise, then the angular acceleration is nonzero.

$$\alpha = \frac{(\omega_2 - \omega_1)}{t}$$

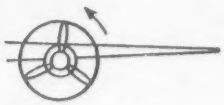
$$\alpha = \frac{(-\omega_1 - \omega_1)}{t} \quad \text{(Since } \omega_2 = -\omega_1\text{)}$$

$$\alpha = \frac{-2\omega_1}{t}$$

Of course, the clockwise direction can be chosen as positive, and counterclockwise is then negative. Angular acceleration has units of radians per second per second, or rad/s^2. Example B demonstrates the use of Equation 5 in solving a problem.

EXAMPLE B: ANGULAR ACCELERATION OF AN AIRPLANE PROPELLER

Given: An airplane propeller initially at rest accelerates at a constant angular acceleration of 420.0 rad/s^2 for 0.5 seconds.



Find: Its final angular velocity.

Solution: The initial angular velocity ω_1 is zero. From Equation 5 —

$$\omega_2 = \omega_1 + \alpha t$$

$$= 0 + (420.0 \text{ rad/s}^2)(0.5 \text{ s})$$

$$\omega_2 = 210.0 \text{ rad/s}$$

or in r/min —

$$\omega_2 = 210.0 \times 60/2\pi$$

$$\omega_2 = 2005 \text{ r/min}$$

An expression relating the angular displacement θ to the angular velocities ω_1 and ω_2 can be obtained from Equation 6 (the corresponding equation borrowed from rectilinear motion).

$$s = \frac{1}{2}(v_1 + v_2)t \qquad (6)$$

where

s = arc length (as in Figure 2).
v_1 = initial linear velocity.
v_2 = final linear velocity.

Angular displacement is found as follows from Equation 6:

$$s/r = (v_1 + v_2)t/2r \quad \text{(Both sides divided by r)}$$

$$\theta = \frac{1}{2}(v_1/r + v_2/r)t \quad (\theta \text{ substituted for } s/r)$$

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t \quad \text{(Since } \omega_1 = v_1/r \text{ and } \omega_2 = v_2/r.\text{)} \quad (7)$$

The final angular velocity ω_2 can be eliminated from Equation 7 by use of Equation 5:

$$\theta = \frac{1}{2}(\omega_1 + \omega_1 + \alpha t)t \quad (\omega_1 + \alpha t \text{ substituted for } \omega_2.)$$

$$\theta = \frac{1}{2}(2\omega_1 + \alpha t)t$$

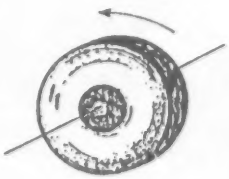
$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 \quad \text{(Multiplied by t/2 inside)} \quad (8)$$

Equation 8 is the counterpart in rotation to the expression $s = v_1 t + 1/2\ at^2$ for linear displacement. Its use is demonstrated in Example C.

EXAMPLE C: ROTATION OF A GRINDSTONE

Given: The angular acceleration of a grindstone is 2.5 rad/s^2. The grindstone starts from rest and attains final velocity after 4.0 seconds.

Find: Its (a) angular displacement and its (b) final angular velocity after 4.0 seconds.



Solution: (a) Equation 8 is used to find θ; the initial angular velocity ω_1 is zero.

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \frac{1}{2}(2.5 \text{ rad/s}^2)(4.0 \text{ s})^2$$

$$\theta = 20.0 \text{ radians}$$

(b) The final velocity is found from Equation 7:

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$$

$$20.0 \text{ rad} = \frac{1}{2}(0 + \omega_2)(4.0 \text{ s})$$

$$\omega_2 = 2(20 \text{ rad})/(4.0 \text{ s})$$

$$\omega_2 = 10.0 \text{ rad/s}$$

This result can also be found by using Equation 5.

Sometimes, the necessity arises to calculate ω_2 from ω_1, α, and θ without knowledge of the elapsed time t. This relation is derived as follows to result in Equation 9.

$$\omega_2 = \omega_1 + \alpha t \quad \text{(Equation 5)}$$

$$t = (\omega_2 - \omega_1)/\alpha \quad \text{(Solving Equation 5 for t)}$$

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t \quad \text{(Equation 7)}$$

$$\theta = \frac{1}{2}(\omega_2 + \omega_1)(\omega_2 - \omega_1)/\alpha \quad \text{(Substituting } \omega_2-\omega_1/\alpha \text{ for t in Equation 7)}$$

$$\theta = \frac{1}{2}(\omega_2^2 - \omega_1^2)/\alpha \quad \text{(Multiplying terms in parenthesis)}$$

$$2\alpha\theta = \omega_2^2 - \omega_1^2 \quad \text{(Multiplying both sides by } 2\alpha)$$

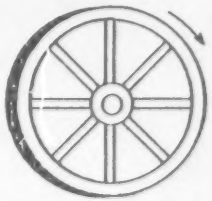
$$\omega_2^2 = \omega_1^2 + 2\alpha\theta \qquad (9)$$

Equation 9 also has an analogue in rectilinear motion: $v_2^2 = v_1^2 + 2as$. Its use is illustrated in Example D.

EXAMPLE D: ROTATION OF A FLYWHEEL

Given: A wheel initially rotating at 4 rev/s decelerates at a rate of 2 rev/s^2.

Find: The number of revolutions it makes before stopping.



Solution: In this problem, the units can be left in rev/s without converting them to rad/s. The final angular velocity ω_2 is zero.

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

$$0 = (4\ \text{rev/s})^2 + 2(-2\ \text{rev/s}^2)\theta$$
$$0 = 16\ \text{rev}^2/\text{s}^2 - (4\ \text{rev/s}^2)\theta$$
$$\theta = (16\ \text{rev}^2/\text{s}^2)/(4\ \text{rev/s}^2)$$
$$\theta = 4\ \text{rev}$$

The parallels between translation and rotation are summarized in Table 1.

TABLE 1. ANALOGY BETWEEN TRANSLATION AND ROTATION

Concept	Translation	Rotation
Displacement	s	θ
Velocity	$v = \frac{s}{t}$	$\omega = \frac{\theta}{t}$
Acceleration	$a = \frac{(v_2 - v_1)}{t}$	$\alpha = \frac{(\omega_2 - \omega_1)}{t}$
Constant Acceleration	$v_2 = v_1 + at$	$\omega_2 = \omega_1 + \alpha t$
	$s = v_1 t + \frac{1}{2} at^2$	$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$
	$v_2^2 = v_1^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

Example E shows how two of the equations describing rotation are solved for the problem of a flywheel.

EXAMPLE E: ROTATION OF A FLYWHEEL

Given: The angular acceleration of a flywheel is 2 rad/s^2. It rotates through 50 radians in a 5-second time interval sometime after it started from rest.

Find: How long the flywheel had been turning just before the beginning of the 5-second interval.

Solution: The angular velocity at the beginning of the 5-second interval is found as follows:

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$50 \text{ rad} = \omega_1(5 \text{ s}) + \frac{1}{2}(2 \text{ rad/s}^2)(5 \text{ s})^2$$

$$\omega_1 = (50 \text{ rad} - 25 \text{ rad})/5 \text{ s}$$

$$\omega_1 = 5 \text{ rad/s}$$

This value is the angular velocity at the end of the time interval between the beginning of rotation and the beginning of the 5-second interval. The initial angular velocity ω_1 at time $t = 0$ is zero since the flywheel started from rest.

$$\omega_2 = \omega_1 + \alpha t$$

$$5 \text{ rad/s} = 0 + (2 \text{ rad/s}^2)t$$

$$t = (5 \text{ rad/s})/(2 \text{ rad/s}^2)$$

$$t = 2.5 \text{ s}$$

Thus, the flywheel had been spinning (from rest) for 2.5 s before it was found to turn through 50 radians in the next five seconds

SUMMARY

"Rotary motion" is the motion of a body turning about an axis. Rotary motion occurs in a turning wheel, the rotating crankshaft of an automobile engine, and a pulley wheel attached to the spinning shaft of an electric motor. Notice the difference between circular and rotary motion. In circular motion, the object as a whole travels along a circular path. In rotary motion, the object simply spins. While a bicycle wheel spinning on a stationary axle undergoes rotary motion, a part of the wheel, say the valve stem of the tire, experiences circular motion.

For rotary motion to be uniform, the object must spin about a fixed axis at a definite rate. The movements of the hands of an electric clock are examples of uniform rotary motion. If either the direction of the axis or the rate of spin varies, the rotary motion is variable. The movements of automobile wheels as the car is driven at different speeds, and the movements of a spinning top as it falls down, are examples of variable rotary motion.

The rate of rotary motion is called "angular velocity." For an object in uniform rotary motion, angular velocity equals the angle through which the object turns divided by the time required for this rotation. Angular velocity may be measured in revolutions per minute. However, it includes not only the rate of rotation, but the direction of the axis of rotation. Thus, angular velocity

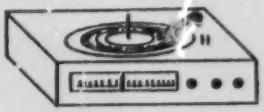
is a vector quantity, represented by a vector parallel to the axis of rotation. The length of the vector indicates the magnitude of the angular velocity.

"Angular acceleration" is the change in angular velocity divided by the elapsed time.

ANGULAR VELOCITY AND ANGULAR ACCELERATION OF A TURNTABLE

MATERIALS

Phonograph turntable.
Stopwatch.
Weights.
Chalk.



PROCEDURE

1. Make a small chalk mark on the rim of the turntable. Set the speed at 45 rpm. Measure the time in seconds required for the chalk mark to pass a reference point of your choosing through ten revolutions. Record this time in the Data Table.

2. Convert the time to minutes by dividing by 60, and record in Data Table. Calculate angular velocity in rpm by dividing angular displacement θ = 10 rev by time in minutes. Convert to rad/s, and record both values. How close is your value to 45 rpm?

3. Repeat Steps 1 and 2 for a speed of 33-1/3 rpm.

4. Reset speed to 45 rpm, and start turntable. Start stopwatch, and turn turntable off simultaneously. Stop watch when turntable comes to a complete stop. Record time in seconds in Data Table.

5. Convert time in seconds to minutes by dividing by 60. Record time in minutes in Data Table. Calculate angular deceleration α of turntable in r/min^2, using measured value of ω_1 in r/min from Step 2. (Remember that the final angular velocity ω_2 is zero.) Calculate α in rad/s^2, using ω_1 in rad/s and time in seconds. Is α positive or negative?

6. Repeat Steps 4 and 5, after placing a weight of 100g on turntable.

7. Explain how this experiment demonstrates the principles explained in the Discussion section.

DATA TABLE
Time required for 10 revolutions at 45 rpm = _____ s
Time required for 10 revolutions at 33-1/3 rpm = _____ s
Time required for turntable to decelerate = _____ s
Time required for weighted turntable to decelerate = _____ s
RESULTS
Time required for 10 revolutions at 45 rpm = _____ min
Angular velocity ω = _____ rpm
Angular velocity ω = _____ rad/s
Time required for 10 revolutions at 33-1/3 rpm = _____ min
Angular velocity ω = _____ rpm
Angular velocity ω = _____ rad/s
Time required for turntable to decelerate = _____ min
Angular deceleration α = _____ r/min^2
Angular deceleration α = _____ rad/s^2
Time required for weighted turntable to decelerate = _____ min
Angular deceleration α = _____ r/min^2
Angular deceleration α = _____ rad/s^2

SELF CHECK

1. State and define the rotational analogue to each of the following:

   Initial velocity, v_1.
   Final velocity, v_2.
   Constant acceleration, a.
   Elapsed time, t.

Displacement, s.

$v_2 = v_1 + at.$

$s = v_1 t + \frac{1}{2} at^2.$

$v_2^2 = v_1^2 + 2as.$

2. Find the linear speed of a point on a record at the needle position (a) at the beginning of the recording, where the radius is 15 cm, and (b) at the end of the recording, where the radius is 7.4 cm, for a phonograph speed of 78 rpm.

3. An automobile has wheels of radius 38 cm and travels at 25 m/s. Find (a) the angular speed of the wheels about the axle; (b) the angular deceleration if the wheels are brought to a stop after 30 turns; (c) the distance traveled during braking. (Hint: the rim speed of the tire is equal to the translational speed of the automobile.)

4. The angular speed of the flywheel of a steam engine is 160 r/min. The wheel comes to rest in 25 hours when the steam is turned off. Find the number of revolutions made by the wheel before it comes to rest.

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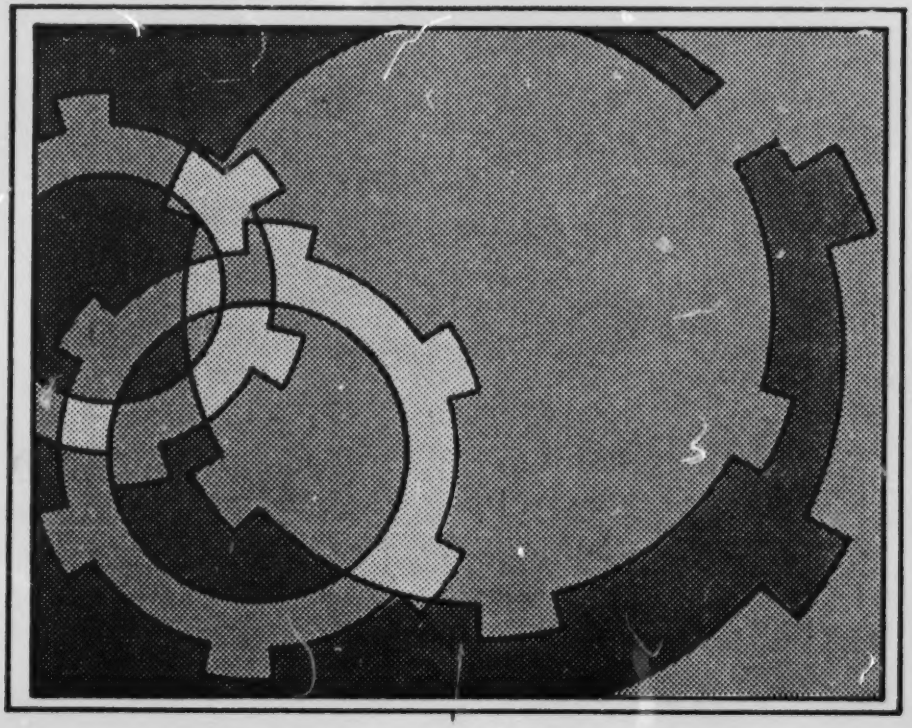
UNIFIED TECHNICAL CONCEPTS

MODULE 3 M 6
RADIAL ACCELERATION
& CENTRIPETAL FORCE



MECHANICAL APPLICATION OF RATE

Technical Education Research Center-Southwest TERC-SW

INTRODUCTION

When rotational motion was studied previously, the concepts of angular displacement, angular velocity, and angular acceleration were used. In order to advance the study of angular motion, the concepts of radial (centripetal) acceleration, contripetal force, and centrifugal force are discussed here. These concepts will be used in studying the causes and effects of circular motion. The operation of governors and centrifugal pumps which depend upon centripetal acceleration and centripetal forces also will be examined. In the laboratory, the student will determine the centripetal acceleration and centripetal force related to a given rotating object.

PREREQUISITES

Student should have completed Concept Module 4-0, "Rate," and Application Module 3M5, "Rotational Motion."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the resulting angular displacement and angular velocity for a rotating body at a specified time, given its initial angular velocity and angular acceleration.
- Define "tangential velocity" of an object in circular motion.
- Sketch on a diagram the radial (centripetal) acceleration and centripetal force acting on a mass in circular motion.
- Calculate the tangential velocity of an object in circular motion, given its lap time and the radius of its path.
- Calculate the radial (centripetal) acceleration of an object undergoing circular motion, given the tangential velocity and radius of the circle.
- Calculate the centripetal force on an object, given its centripetal (radial) acceleration and its mass.

- Describe, using a diagram, how the following devices work:

  A mechanical governor.
  A centrifuge.
  A centrifugal pump.

- Given the appropriate equipment, determine the centripetal acceleration and centripetal force of a rotating object.

DISCUSSION

The main variables introduced in studying angular motion are angular displacement θ, angular velocity ω, and angular acceleration α. Constant, or average, angular velocity was defined as "the change in angular displacement θ, divided by the time t, during which the change takes place." Constant, or average, angular acceleration was defined as "the change in angular velocity ω, divided by the time t, during which the change takes place."

Angular terms and equations are given in Table I, along with the analogous linear terms and equations.

TABLE I: ANGULAR TERMS & EQUATIONS RELATED TO LINEAR ONES

Angular Term or Equation		Linear Term or Equation
θ	DISPLACEMENT	x
ω	SPEED	v
α	ACCELERATION	a
$\omega = \omega_o + \alpha t$		$v = v_o + at$
$\omega^2 = \omega_o^2 + 2\alpha\theta$		$v^2 = v_o^2 + 2ax$
$\theta = \frac{\omega_o t + \alpha t^2}{2}$		$x = \frac{v_o t + at^2}{2}$

Some of the angular equations in Table I are employed to solve problems in Examples A and B.

EXAMPLE A: CONSTANT ANGULAR VELOCITY

Given: A flywheel rotates at a constant angular velocity of 5 rev/s for 5 s.

Find: The angular displacement in rev and degrees.

Solution: Since the angular velocity is constant, the angular acceleration is zero.

$$\theta = \frac{\omega_o t + \alpha t^2}{2}$$

$$= (5 \text{ rev/s})(5 \text{ s}) + (0)$$

θ = 25 rev

Since there are 360 degress in one rev —

θ = (25 rev)(360 deg/rev)

θ = 9000 deg

EXAMPLE B: ANGULAR ACCELERATION

Given: A rotating shaft starts from rest with a constant angular acceleration of 20 rev/s^2.

Find: The angular velocity and angular displacement after 10 s.

Solution: The angular velocity is given by —

$$\omega = \omega_0 + \alpha t$$

$$= (0) + (20 \text{ rev/s}^2)(10 \text{ s})$$

$$\omega = 200 \text{ rev/s}$$

The angular displacement is given by —

$$\theta = \frac{\omega_0 t + \alpha t^2}{2}$$

$$= \frac{(0) + (20 \text{ rev/s}^2)(10 \text{ s})^2}{2}$$

$$\theta = 1000 \text{ rev}$$

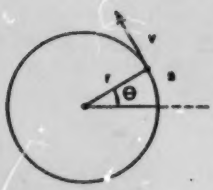


FIG. 1

Figure 1 depicts (1) a particle traveling in a circular path which has rotated through an angle θ, and (2) the corresponding tangential displacement s. A simple relation between θ, s, and the radius r is given by Equation 1.

$$s = r\theta \qquad (1)$$

where

s = tangential displacement.
r = radius.
θ = angle of circular path in radians.

Recall that the angle corresponding to a complete circle is defined as 2π radians. Then, according to Equation 1, the path length, or circumference, is indicated by Equation 2.

$$c = 2\pi r$$

where
c = circumference.

"Tangential velocity" is defined as "the tangential displacement divided by time," and is given by Equation 2.

$$v = \frac{s}{t} \qquad (2)$$

where
v = tangential velocity.
s = tangential displacement.
t = time.

From Equation 1, the tangential velocity of Equation 2 takes the form in Equation 3.

$$v = \frac{(r\theta)}{t}$$

$$= r\left(\frac{\theta}{t}\right)$$

$$v = r\omega, \text{ since } \omega = \frac{\theta}{t} \qquad (3)$$

where
ω = angular velocity.

This velocity is tangent to the circle, as shown in Figure 1.

If a particle is traveling in a circular path and has zero angular acceleration, then its tangential velocity will have the same magnitude at all points along its path; however, the direction of this tangential velocity will be changing continually.

Figure 2a shows a particle of mass m traveling in a circular path on the end of a string. If the string breaks, as in Figure 2b, the particle will follow a straight-line path, tangent to the circle and in the plane of the circle, with a speed given by Equation 2.

(a.) (b.) (c.)

FIG. 2

Recall from Newton's first law that a particle will travel in a straight line, at constant speed, unless a force acts upon it. Because the mass in Figure 2a is not traveling in a straight line, it must have a force acting upon it in order for its motion to change. In the case of a circular path, the mass can be traveling at a constant tangential speed. Thus, the magnitude of the vector velocity does not change; rather, its direction changes.

Newton's second law states that the net force acting upon an object is equal to the product of its mass times its acceleration.

$$F = ma \qquad (4)$$

where

F = force acting upon an object.
m = mass.
a = acceleration.

Figure 2c shows the force that must be acting upon the mass to produce a circular path. (The force must be constant in magnitude and always directed toward the center.) Since the acceleration must be in the same direction as the force, it also acts toward the center of the circle. This force, necessary to produce circular motion, is called the "centripetal force." ("Centripetal" means "center seeking.") The corresponding acceleration is called the "centripetal," or "radial" acceleration.

In Figure 2a, the force and acceleration act along the string in order to keep the mass traveling in a circular path. If the string breaks as in Figure 2b, the force suddenly becomes zero; and according to Newton's first law, the mass must move off tangentially in a straight line.

Newton's second law is now written for centripetal force as Equation 5.

$$F_c = ma_c \qquad (5)$$

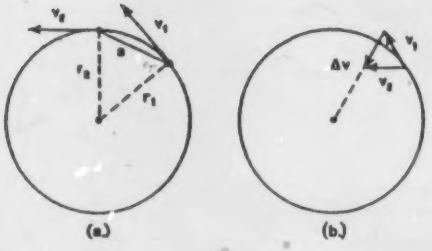


FIG. 3

where

F_c = centripetal force.
a_c = centripetal acceleration.

An equation for centripetal acceleration in Equation 5 can be found by a consideration of Figure 3. Figure 3a illustrates a mass in circular motion at two different times, separated by a distance s and having corresponding tangential velocities

v_1 and v_2. Figure 3b shows the two tangential velocities added together as vectors to result in the change in velocity Δv, which is directed toward the center. The triangles (r,s,r) and (v_1,Δv, v_2) are similar, as indicated by Equation 6.

$$\frac{s}{r} = \frac{\Delta v}{v_1} \tag{6}$$

The magnitudes of the velocities v_1 and v_2 are equal ($v_1 = v_2 = v$). Δv is solved for by Equation 7.

$$\Delta v = \frac{vs}{r} \tag{7}$$

The acceleration a_c is the change in velocity divided by the time, or $a_c = \Delta v/t$; therefore, Equation 7 can be used to obtain $a_c = vs/rt$ or $a_c = (v/r)(s/t)$.

By use of the relationship $v = s/t$, centripetal acceleration can be written in the form of Equation 8, which gives the simple relation between the tangential velocity and the centripetal acceleration.

$$a_c = \left(\frac{v}{r}\right) v$$

$$a_c = \left(\frac{v^2}{r}\right) \tag{8}$$

Centripetal force can now be written as in Equation 9.

$$F_c = ma_c = \frac{mv^2}{r} \tag{9}$$

Recall that v is the tangential velocity as expressed in Equation 3. The centripetal acceleration given by Equation 8 also is called the "radial acceleration."

EXAMPLE C: CENTRIPETAL ACCELERATION AND FORCE

Given: A 3000-lb car travels at constant speed on a circular track having a radius of 200 ft. The car requires 15 s to complete one lap.

Find: Find (a) the tangential velocity, the (b) centripetal acceleration and the (c) centripetal force.

Solution: (a) The tangential velocity can be found by use of these equations:

$$v = \frac{2\pi r}{t}$$

$$= \frac{2\pi\ (200\ \text{ft})}{15\ \text{s}}$$

$$v = 83.8\ \text{ft/s}$$

(b) Equation 8 gives the centripetal acceleration:

$$a_c = \frac{v^2}{r}$$

$$= \frac{83.8\ \text{ft/s}^2}{200\ \text{ft}}$$

$$a_c = 35.1\ \text{ft/s}^2,$$

directed toward the center of the circular path.

(c) The centripetal force is given by Equation 9:

$$F_c = ma_c = (w/g)\ a_c$$

$$= \left(\frac{3000\ \text{lb}}{32\ \text{ft/s}^2}\right)(35.1\ \text{ft/s}^2)$$

$$F_c = 3291\ \text{lb},$$

directed toward the center of the circular path.

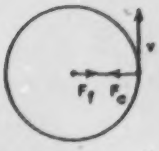


FIG. 4

Consider again the example of a mass traveling in a circular path on the end of a string, as illustrated in Figure 4. The centripetal force F_c acts inward on the mass toward the center of the circle. Newton's Third Law — "For every action there is an equal and opposite reaction" — indicates that there is an outward acting force F_f, called the "centrifugal force," which acts outward upon the center support. These two forces, centripetal (inward) and centrifugal (outward), are equal in magnitude, opposite in direction, and act upon different objects. If the string were being held by a hand, the centrifugal force would be acting outward on the hand toward the mass.

FIG. 5

Consider a car traveling in a circular path on a level road, as depicted in Figure 5. The centripetal acceleration and force act inward toward the center upon the tires of the car. In this case, it is a force caused by friction between the tires and road. The centrifugal force is the frictional force produced by the tires acting outward upon the road.

When a car travels in a circular path, a person in the car seems to experience an outward force; that is, a force tending to throw him or her outward from the center of the circle.

Actually, the seat exerts an inward force upon the person, causing him to travel in a circular path. The person tends to travel in a straight line, but the centripetal force acting inward (provided by the seat) causes him to travel in a circular arc.

Consider two particles m and M, which have the same size and which are suspended in a uniform fluid in a centrifuge (Figure 6). The density of M is greater than the density of m. As the centrifuge spins, both experience the same centripetal acceleration, $a_c = v^2/r$, since the radius and the tangential velocities are the same. The centripetal force upon M, necessary to hold M in place, is given by Equation 10.

$$F_{cM} = Ma_c \qquad (10)$$

The centripetal force for M must be greater than that to hold m in place, since M > m, as indicated by Equation 11.

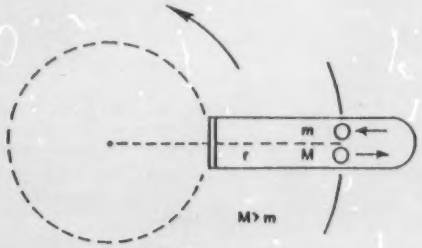


FIG. 6

$$F_{cm} = ma_c \qquad (11)$$

However, the fluid exerts the same fluid force on both masses, since they are of the same size. Because of this, the masses will separate, with the larger mass M moving further outward in the tube than the smaller mass m. The centrifuge uses this principle to separate particles of varying densities. The particles of greater density move toward the bottom (farther from the center) of the tube.

If the same particles are placed on ends of identical springs and swung in a circle, the one with the greater mass will stretch its spring more because it requires a greater restoring (centripetal) force. The larger mass will rotate with a greater radius, as in the centrifuge.

Another application of centripetal force in circular motion is illustrated in Figure 7. The fluid enters the pump at the center inlet and is given a spinning motion. Its natural tendency is to move in a straight line, tangent to its circular motion, unless an inward centripetal force keeps it in a circular path. Since there is no string or spring tied to each particle, there is no inward force. Thus, the fluid flows outward along lines tangent to its circular motion at each point.

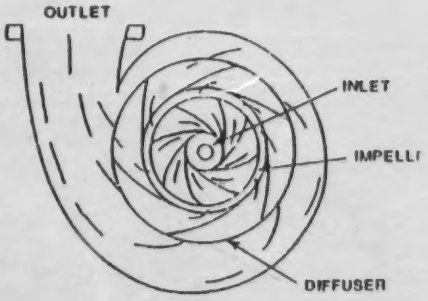


FIG. 7

Because the fluid moves outward, it is called a "centrifugal pump." Centrifugal means "center fleeing."

SUMMARY

Centripetal force is the inward force upon an object moving in a circular path. It is the radial force exerted by the constraining system required to keep the object moving in its curved path around a central point. Whenever rotational motion occurs, centripetal force is present because such a force is necessary to change the direction of object motion from the straight line that it normally would follow to a curved path. Before the magnitude of the force required to keep a given object moving in a circle at a certain speed can be determined, the acceleration of the object must be found. This radial, or centripetal, acceleration is the change in velocity divided by the time interval to express a rate.

CENTRIPETAL ACCELERATION & CENTRIPETAL FORCE

MATERIALS

Centripetal force apparatus including weights and weight hanger (Sargent-Welsh #0930 or equivalent).
Meter stick.
Stop watch or timer.

PROCEDURE

In this experiment, the student will determine the centripetal acceleration of a rotating object and find the corresponding centripetal force by two different methods. The percentage difference between these forces will be calculated.

Figure 8a displays the centripetal force apparatus as it is rotated. The mass m swings outward due to the rotation, but the spring supplies the inwardly-directed centripetal force F_c necessary to keep the mass traveling in a circular path. The greater the rotational speed, the further out the mass swings. Figure 8b shows the stationary spring being stretched the same distance as in Figure 8a, but due to a force supplied by weight W. For equilibrium to exist in Figure 8b, F must equal W.

Since the spring is displaced the same amount in Figure 8a and 8b, the two forces F_c and F must be equal.

1. Rotate apparatus as shown in Figure 8a. Adjust speed until the mass m swings directly above radius indicator rod. Practice will allow rate to be held constant, so that the mass passes directly over the indicator rod on each revolution. Use the timer to determine the total time T_t for 30 rotations of mass, and record this in Data Table 1. Calculate the average period $T = T_t/30$ for each rotation. Record this value. Repeat this process three times, and find the average period for these three trials.

2. Measure and record radius r of rotational motion, and record this value.

3. Attach weight hanger as shown in Figure 8b. Place additional weights on hanger until rotating mass is held over radius indicator bar. Record this mass value M (weight hanger plus

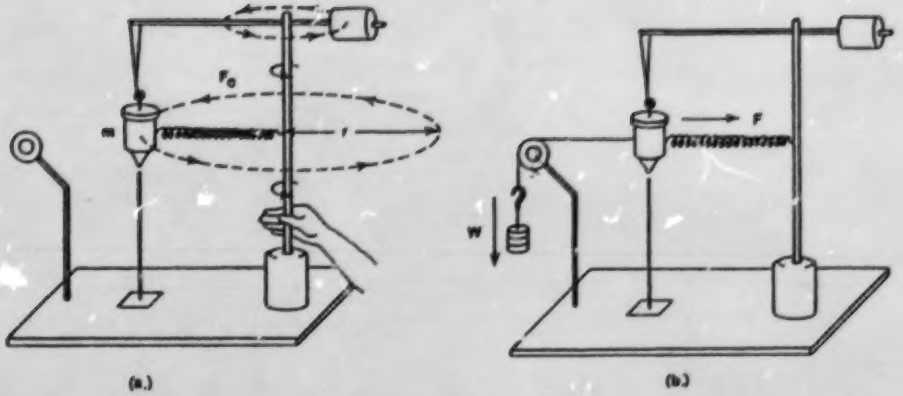


FIG. 8

added weights) in Data Table 2. Determine corresponding weight W = Mg and record this value.

4. Calculate velocity of rotating mass illustrated in Figure 8a by dividing the total distance traveled ($2\pi r$) by the average period (T) from Data Table 1. Since the tangential velocity is the total distance traveled, divided by the time, then $v = 2\pi r/T$. Record the values of T, r, and v in Data Table 2.

5. The centripetal acceleration for the rotational motion now can be calculated from the equation $a_c = v^2/r$ and recorded in Data Table 2. Calculate the centripetal force acting upon the rotating mass m from the equation $F_c = ma_c$ and record in Data Table 2.

6. The force F = W and the centripetal force F_c should be the same, since they produce an equal elongation of the spring. Determine the percentage difference in the measured values of these two forces. Record this value.

7. Repeat steps 1 through 6 for a different tension in the spring. This repetition can be achieved by changing the position of the radius indicator rod and by correspondingly changing the position of the rotating crossbar.

8. How would a radius of greater length affect the results?

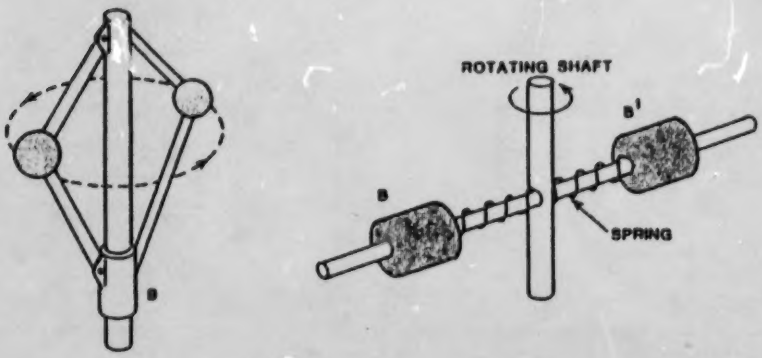
DATA TABLE 1				
Spring Tension Setting	Trial No.	Time for 30 rev T_t (s)	Period T (s)	Radius r (m)
Tension 1	1			
	2			
	3			
	Average			
Tension 2	1			
	2			
	3			
	Average			

DATA TABLE 2		
Spring Tension Setting	1	2
Period	T = s	T = s
Radius of rotation	r = m	r = m
Mass to elongate spring	M = kg	M = kg
Weight to elongate spring	W = Mg = N	W = Mg = N
Speed of rotating body	v = m/s	v = m/s
Centripetal acceleration	$a_c = v^2/r$ = m/s^2	$a_c = v^2/r$ = m/s^2
Mass of rotating object	m = kg	m = kg
Centripetal Force	$F_c = ma_c$ N	$F_c = ma_c$ N
Percentage difference between F_c and W	%	%

SELF CHECK

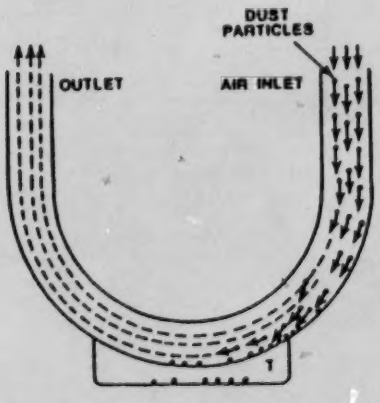
1. A merry-go-round at a park is set into motion in such a manner that it completes one revolution each 3 seconds. A child weighing 50 lbs sits on the edge of the merry-go-round a distance of 6 ft from its center. Calculate (a) the tangential velocity of the child, (b) the centripetal acceleration, and (c) the centripetal force acting upon the child.

2. Two types of governors used for controlling the speed of devices are shown. The cylinders B and B' are free to slide as the rotational speed changes. Explain, using the ideas of centripetal acceleration and centripetal force, how the governors can be used to control the speed of a rotating object.

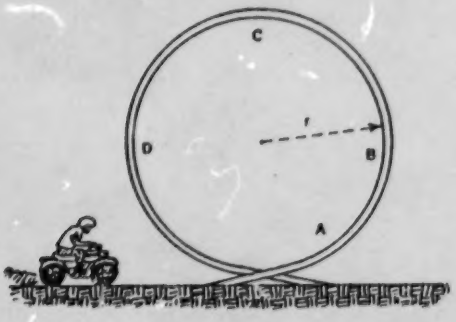


3. What is the difference between centripetal and centrifugal force?

4. A "cyclone-dust-remover" is shown in the adjoining figure. Explain how it can be used to collect dust particles in the collector T, which often contains water or a water spray.



5. A motorcyclist loops-the-loop as shown in the adjoining figure. What is the minimum velocity he must have at C in order not to lose contact with the surface? Take r = 16 ft and g = 32 ft/s^2. (Hint: At point C, F = mg = mv^2/r.)



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UNIFIED TECHNICAL CONCEPTS

MODULE 3 F 1

FLOW RATE MEASUREMENT WITH A VENTURI METER

FLUID APPLICATION OF RATE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Together with measurements of temperature and pressure, flow rate of fluids is one of the more important measurements required in many industrial processes. Manufacturing plants use devices to measure fluid flow rates to establish the proper ratio of materials needed in continuous oil and chemical processing and to control temperature by means of cooling systems.

This module will present operating principles of a venturi flow meter, a typical device used in industry for accurate flow rate measurements. An experiment is included to give the student hands-on experience in the use of a laboratory model of a venturi tube.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

To complete this module, the student should be able to:

- Sketch and label the four aspects of a venturi meter and describe in a short paragraph how it functions.
- Calculate fluid velocity given appropriate conditions and equations.
- Using a venturi meter, mercury manometer and associated items of equipment, measure fluid flow in a pipe system and describe the results.

DISCUSSION

In Unified Concept Module 3-0, "Rate," it was explained that fluid velocity by itself is not adequate to describe flow rate because size of the container carrying the fluid must also be considered. If the physical dimensions of a pipe or tube in a hydraulic system are considered, fluid velocity can be used as a measure of flow rate in the system. The relationship between flow rate and fluid velocity is called the flow rate equation and is given in Equation 1.

$$\text{Volume flow rate} = (\text{cross section area}) \times (\text{fluid velocity}). \quad (1)$$

Consider a pipe of variable cross section completely filled with a moving liquid. Since liquids are difficult to compress, the volume flow rate in gal/min, for example, remains constant throughout the length of the pipe, even though diameter of the pipe changes from one region to another. As seen from Equation 1, if the volume flow rate along a pipe at two positions, with different diameters, remains constant, fluid velocity must change. Thus fluid velocity is higher where diameter is smaller and lower where diameter is larger. Example A and Equation 1 show how a constriction in a pipe increases velocity of the fluid. Equation 1 is used to solve for final fluid velocity.

VENTURI METER

A venturi meter is a device used by industry to measure flow rate of a liquid or gas. For industrial use, it is built of cast metal and machined to close tolerances by the manufacturer. Size may range from small models with dimensions of a few inches to large models with 60-inch diameters and 40-ft lengths, having flow capacities of 80,000 gallons per minute. Large model venturi tubes are built in several sections and bolted together. A typical industrial venturi tube is shown in Figure 1.

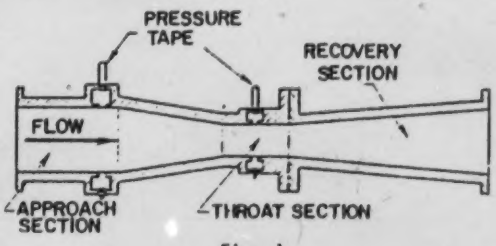


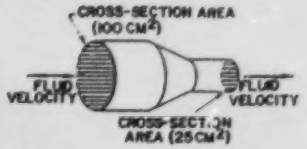
Fig. 1

The venturi meter is a highly accurate instrument, expensive, when carefully manufactured and calibrated. Its smooth internal design allows any

EXAMPLE A: EFFECT OF PIPE DIAMETER CHANGE ON FLUID VELOCITY

Given: The pipe shown, with fluid entering at 2 cm/s through a cross sectional area of 100 cm^2.

Find: Fluid velocity when the pipe tapers to a cross sectional area of 25 cm^2.



Solution:

(a) At left end of the pipe with large diameter:

$$\text{Volume flow rate} = \left(\frac{\text{cross sectional}}{\text{area}}\right) \times \left(\frac{\text{fluid}}{\text{velocity}}\right)$$

$$= (100\ cm^2) \times (2\ cm/s)$$

$$\text{Volume flow rate} = 200\ cm^3/s$$

Since the liquid is not compressed while flowing, volume flow rate at the small end equals volume flow rate at the large end. Fluid velocity at the small end can then be calculated by using Equation 1.

Rearranging Equation 1.

(b) At right end of the pipe with small diameter:

$$\text{Fluid velocity} = \frac{\text{volume flow rate}}{\text{cross section area}}$$

$$= \frac{(200\ cm^3/s)}{25\ cm^2}$$

$$\text{Fluid velocity} = 8\ cm/s$$

Fluid velocity at the small end is four times faster than its velocity at the large end. This is to be expected, since cross sectional area was reduced by four times, and yet volume flow rate must be the same at the large and small ends of the pipe.

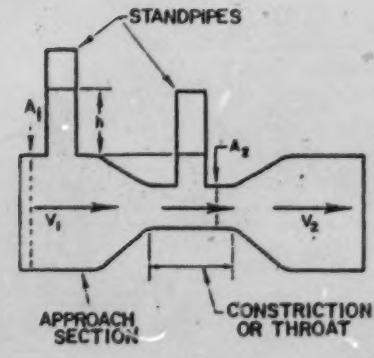


Fig. 2

solid particles to pass through the liquid without affecting its accuracy.

A schematic of a simple venturi meter is shown in Figure 2. The tube has cross sectional area A_1 and A_2. When fluid moves through the meter, the constriction causes velocity to increase, as in Example A. It is also true, although not proven yet, that an increase in velocity causes an accompanying decrease in fluid pressure. Thus, pressure of the fluid in the constricted section of the venturi meter at A_2 is less than pressure in the fluid at A_1. This phenomenon, which relates fluid pressure and fluid velocity, is known as Bernoulli's principle.

It is Bernoulli's principle that leads to the

curved cross sectional shape of airplane wings. The curved top of an airplane wing produces a higher wind velocity above the wing than below it. The higher velocity of wind above the wing causes a lower pressure above the wing than below, thereby producing lift on the entire wing surface.

Gages or standpipes in the approach section of the venturi meter and in its throat make it possible to measure pressure difference caused by the constriction. When standpipes are used, the measured height difference between the columns of liquid permits the fluid velocity in the approach section to be calculated. Difference in heights of the columns, h, is usually measured with a pressure-measuring device such as a U-tube manometer.

When using a venturi meter to determine fluid flow rate, cross sectional areas of the throat and of the approach section must be obtained from the manufacturer or carefully measured. Fluid velocity in the approach section may then be calculated with Equation 2.

$$V_1 = \sqrt{\frac{2gh}{\left[\frac{A_1}{A_2}\right]^2 - 1}} \qquad (2)$$

where

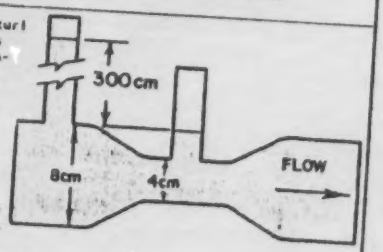
V_1 = velocity of fluid in pipe.
A_1 = cross sectional area of approach section.
A_2 = cross sectional area of throat section.
g = acceleration due to gravity (9.8 m/s^2).
h = height difference of mercury in manometer tubes.

Example B illustrates use of this equation to calculate flow rate.

EXAMPLE 8: MEASURING FLUID FLOW WITH A VENTURI METER

Given: An 8-cm (diameter) venturi reduced to 4-cm diameter at its constriction; pressure differential is 300 cm.

Find: Flow rate when water is flowing.



Solution:

(a) $$V_1 = \sqrt{\frac{2g \times h}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= \sqrt{\frac{2\ (9.8\ m/s^2)(3.00\ m)}{\left(\frac{50.26\ cm^2}{12.57\ cm^2}\right)^2 - 1}}$$

$$= \sqrt{\frac{58.8\ \frac{m^2}{s^2}}{14.99}}$$

$$= \sqrt{3.92}$$

$V_1 = 1.98$ m/s

Fluid velocity in the approach section is 1.98 m/s.

(b) Volume flow rate at the large end may now be found by using the flow equation:

Volume flow rate = cross sectional area x fluid velocity

= 50.26 cm^2 x 1.98 m/s

= 0.005026 m^2 x 1.98 m/s

Volume flow rate = 0.00995 m^3/s

Recall: g = acceleration of gravity, which is 9.8 m/s^2, and 300 cm x 1 m/100 cm = 300 m.

Recall:
Area = π x radius^2.
$A_1 = \pi\ (4\ cm)^2$.
$A_1 = 50.26\ cm^2$.
$A_2 = \pi\ (2\ cm)^2$.
$A_2 = 12.57\ cm^2$.

$$50.26\ cm^2 \times \frac{1\ m}{100\ cm} \times \frac{1\ m}{100\ cm} = 0.005026\ m^2.$$

MATERIALS

Glass venturi tube (18 cm long, 18 mm in diameter, tube narrows at center and has three outlet tubes at center. Ends of tube are flared.) (Suggested sources: Science Kit Inc. or Central Scientific Company.)
Two lengths of 3/4-in automobile heater hose.
Glass tubing for shaping manometer of appropriate commercial unit.
Rubber tubing (1/4-in inside diameter) for connections to outlet tubes of venturi.
Meter stick or metric scale.
Circular clamp for 3/4-in hose.
Two large containers (5-gallon buckets or trash cans).
Two platform scales graduated in kilograms (bathroom scales graduated in pounds may be used).
Stopwatch.

PROCEDURE

1. Arrange venturi tube, closed U-tube manometer and associated equipment as shown in Figure 3.
2. Measure inside diameter of outlet end of exit hose; record this diameter in data table.
3. Place exit hose in excess water container; slowly turn on faucet, checking system for leaks. Repair any leaks if necessary.
4. Slowly turn on faucet; adjust water flow such that a 1-cm difference in mercury level of manometer is observed (h).
5. Record initial scale reading (with empty collection container on scale) in data table.
6. Simultaneously, switch exit hose to collection container and start timer. After a time interval in excess of 2 minutes, stop timer while simultaneously switching exit hose to excess water container. (Anticipate this action to minimize mess that could occur.)

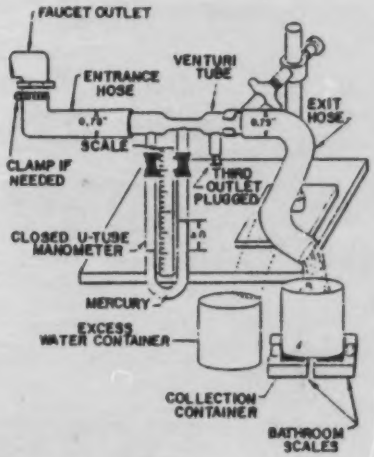


Fig. 3

7. Record total mass readings of scales and time elapsed (from timer) in data table.
8. Empty collection containers and excess water container as needed.
9. Repeat steps 4, 5, 6 and 7 for mercury level differences of 2, 3 and 4 cm on manometer. Record results from each trial in data table.
10. Complete calculations necessary to fill in data table.
    a. Calculate inside cross sectional area of exit hose; record in data table.
    b. Calculate water added to collection container during each trial (kg at end of trial minus kg at start of trial). If scales are graduated in pounds, convert to kg.
    c. Calculate mass flow rate, volume flow rate and fluid velocity (using Equation 1) for each trial in experiment. Record results in data table.

DATA TABLE								
Trial	Manometer Mercury Level Difference (cm)	Collection Time (s)	Scale Reading			Mass Flow Rate (kg/s)	Volume Flow Rate (m^3/s)	Fluid Velocity (m/s)
			Start of Trial (kg)	End of Trial (kg)	Water Added During Trial (kg)			
1	1							
2	2							
3	3							
4	4							

Inside diameter of exit hose _____ m.

Cross sectional area of exit hose _____ m^2.
(Area = πr^2.)

SELF CHECK

1. Sketch a venturi meter; label pressure tape, recovery section, approach section, throat section, flow and explain how it is used to determine flow rate.
2. Using Equation 1, calculate ratio of the approach velocity to the throat velocity in a venturi if inside diameter of the approach is 1.72 cm and inside diameter of the throat is 0.65 cm.
3. Using Equation 2, calculate fluid flow rate using the dimensions given in No. 2 and a manometer pressure level difference (h) of 6 cm.
4. Examining the completed Data Table, observe trial-to-trial behavior of the mercury level difference and velocity of the fluid:
   a. Based on observations, check the answer that describes how fluid velocity (V) changed in the pipe as manometer pressure difference (h) increased.
      _____ (1) Velocity increased.
      _____ (2) Velocity decreased.
      _____ (3) No consistent trend was observed.
   b. If a trend was observed, test data to see if,
      _____ (1) h is directly proportional to V.
      _____ (2) h is directly proportional to V^2.
      _____ (3) h is inversely proportional to V.
      _____ (4) h is inversely proportional to V^2.

   (The test can most easily be done by plotting h in turn versus V, V^2, $1/V$, $1/V^2$. Which yields a straight line?)

   State results of this test in sentence form.

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See especially Chapter 19.

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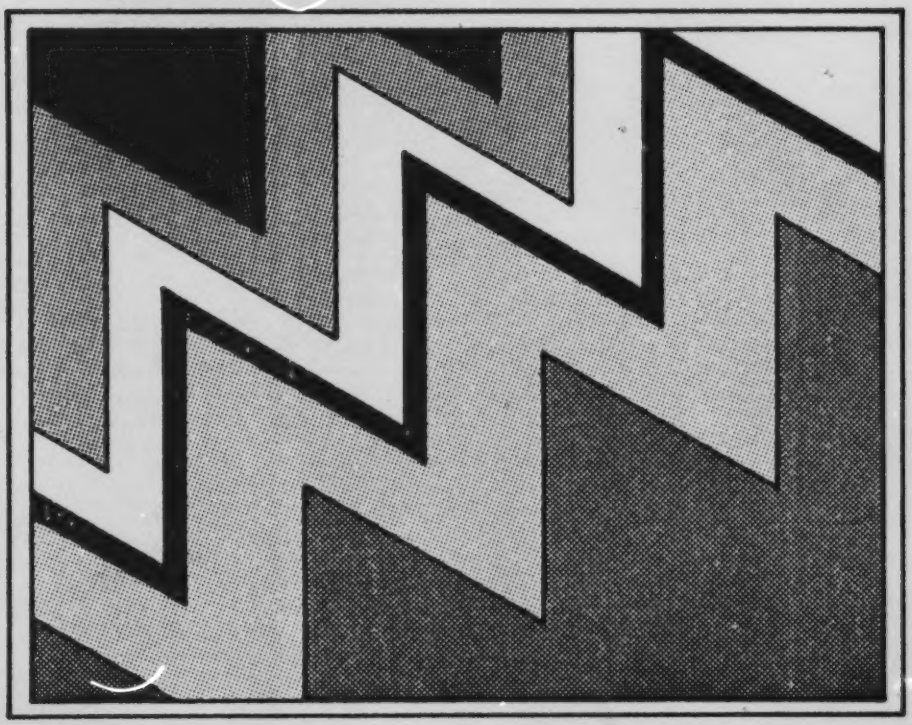
UNIFIED TECHNICAL CONCEPTS

MODULE 3 E 1

ELECTRIC CURRENT



ELECTRICAL APPLICATION OF RATE

Technical Education Research Center - Southwest TERC - SW

INTRODUCTION

The most fundamental "rate" or ratelike quantity that must be understood in relation to electrical systems is the concept of current, the rate at which the displacementlike quantity "charge" is moved from one point to another. Direct current in an electrical circuit is the rate at which charge moves past some point in the circuit when a constant electrical potential is applied. Alternating current is the rate of charge flow back and forth past a point in the circuit when an alternating voltage is applied.

In this module, the student will examine the concepts of direct and alternating current and in the laboratory will use a d.c. ammeter and an a.c. ammeter to measure electric currents.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the below terms:

  Current.
  Direct current.
  Alternating current.
  Ampere.
  Electron flow.
  Conventional current flow.

- Given any two of the following three factors, find the third:

  Current.
  Charge.
  Time.

- Draw and label a graph showing a.c. current as a function of time.

- Given a d.c. ammeter and a direct current source, measure the current within ±0.1 amps.
- Using a galvanometer and a direct current source, determine the direction of flow of the current.

DIRECT CURRENT

In an electrical conductor, some electrical charges — called "electrons" — are free to move in the material. When the poles of an electromotive force or battery are connected by a conductor, these free electrons respond by moving away from the point of negative potential toward the point of positive potential. The terminal of negative potential, called the "cathode," acts as a source of additional electrons; and the terminal of positive potential, called the "anode," acts as a "sink" that removes electrons from the conductor.

As long as an electrical potential difference exists, the electron motion through the conductor will continue. The conducting material itself has no net change in charge. For every electron that enters the conductor from the negative terminal, one electron leaves the conductor at the positive terminal.

To indicate the rate at which current flows through a conductor, a meter must "count" the electrons that pass a given point in some time interval. This process can be visualized by imagining a plane surface within the conductor perpendicular to the electron flow (perpendicular to the conducting wire) as shown in Figure 1.

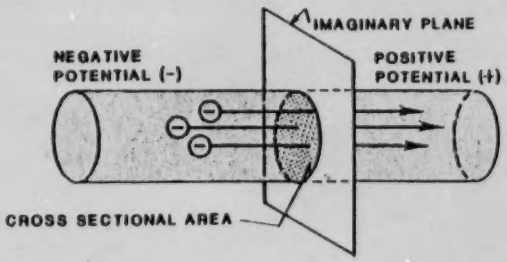


FIG. 1

When a charge Q passes through the surface in a time t, the current is defined by Equation 1 as follows:

$$I = \frac{Q}{t} \qquad (1)$$

where

I = current.
Q = charge.
t = time.

Direct current through a conductor is the charge per unit time that passes in one direction through a plane perpendicular to the applied electric field.

If the charge is measured in coulombs and the time is measured in seconds, resulting current is in "amperes." An ampere

is defined as one coulomb per second. Examples A and B show the use of Equation 1 for solving problems.

EXAMPLE A : CURRENT CALCULATION

Given: 2.4 C (coulombs) passing through a copper wire in 0.6 seconds.

Find: Current.

Solution:

$$I = \frac{Q}{t}$$

$$= \frac{2.4\ C}{0.6\ s}$$

$$I = 4.0\ \frac{C}{s} = 4.0\ A$$

The current is 4.0 amperes.

EXAMPLE B : CHARGE CALCULATION

Given: A current of 62.5 amps flows for 16 ms.

Find: How much charge passes through the conductor.

Solution:

$$I = \frac{Q}{t}$$

$$Q = It$$

$$= (62.5\ amps)(16\ ms) \ldots (ms\ is\ equal\ to\ 10^{-3}\ s)$$

$$= (62.5\ \frac{C}{s})(16 \times 10^{-3}\ s)$$

$$Q = 1.0\ C$$

A charge of 1.0 coulomb passes through the conductor.

DIRECTION OF CURRENT FLOW

This module has described direct current in terms of electron flow. In metal conductors, the charge carriers are the electrons (negative carriers), which move from a point of negative potential toward a point of more positive potential.

In some semiconductor materials, in ionized gases, and in

many solutions, charge is transferred by positive-charge carriers. Positive-carriers result from the removal of an electron at the positive terminal. Such positive-charge carriers move toward the negative terminal, where they receive an electron and become neutral. The result is the same as with electron flow: one unit of charge is transferred through the conductor.

The direction of current flow is somewhat confusing because, at the time electricity was discovered, the positive charge was thought always to be the moving particle. Some texts used in the study of electricity use "conventional current" direction, rather than electron flow direction. The direction of conventional current flow is from positive to negative, and the charge carriers are assumed to be positive. The direction of electron flow is from negative to positive, and the charge carriers are negative electrons.

Both conventional current and electron current are shown in Figure 2. The magnitudes in both directions are the same; only the directions are different.

ALTERNATING CURRENT

Direct current is the rate of charge flow that results when a constant voltage difference is maintained across an electrical conductor. If the voltage changes, the current also changes.

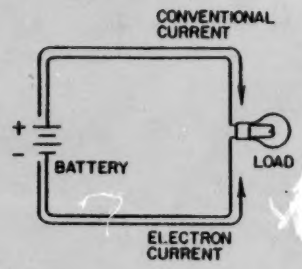


FIG. 2

Alternating current (a.c.) flows first in one direction and then in the other, and results in a circuit when the polarity of the voltage source alternates as shown in Figure 3. At time t_0 in this figure, the applied voltage is zero; and no current flows, as is indicated by t_0 of Figure 4. At t_1 a maximum positive voltage is applied to one end of the conductor, while the other end remains at 0 volts. This voltage difference produces the electron flow shown by t_1 of Figure 4. At t_2 both voltage and current are again zero, and at t_3 an applied voltage of $-V_{max}$ causes a current flow in the opposite direction. At t_4 the voltage and current are both zero again. This waveform repeats at a frequency of 60 cycles/per second in ordinary household circuits.

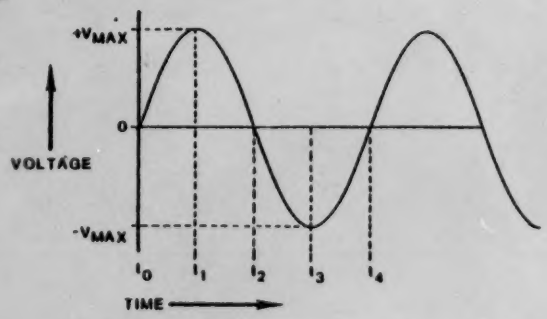


FIG. 3

SUMMARY

The rate at which charge flows through an electrical conductor is called the "current," which is measured in amperes. The "direction of current flow" is defined as the direction in which a positive charge would move for "conventional current" or the direction of motion of a negative charge (electron) for "electron current." "Direct current" (d.c.) is the current flow in a single direction that results when a constant voltage is applied to a circuit. "Alternating current" (a.c.) is the current flow in alternating directions that results when the polarity of the voltage source alternates.

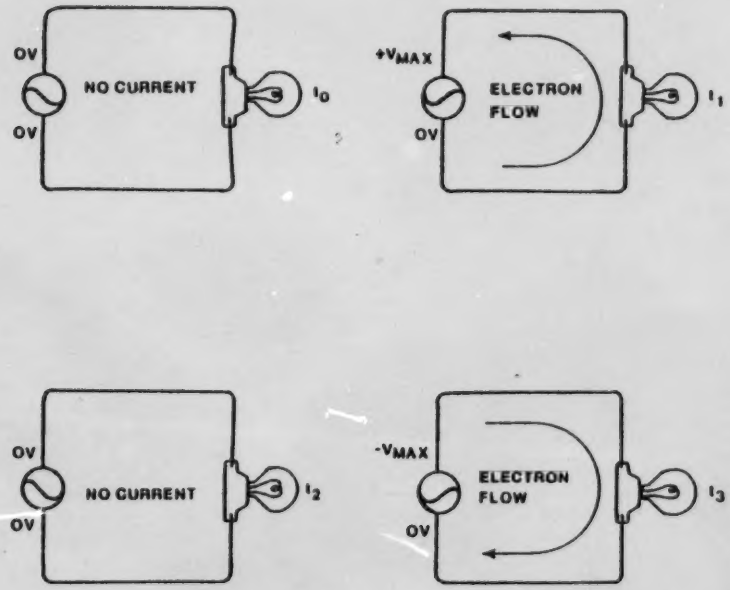


FIG. 4

CURRENT MEASUREMENT

MATERIALS

12-volt battery or d.c. power supply.
12-volt lamp and socket.
d.c. ammeter.
Clip leads.
Knife switch.

PROCEDURE

1. Connect circuit as shown in Figure 5. Notice that positive terminal of ammeter is connected to positive terminal of battery, and that negative terminal of ammeter is connected through lamp to negative terminal of battery. If polarity of meter is reversed, needle will be driven backward and can damage meter mechanism.

2. Close switch; observe both lamp and meter.

3. Record current in Data Table; open switch. Switch should be closed for short periods of time only to conserve battery's energy.

4. Disconnect circuit; reconnect it as shown in Figure 6.

5. Close switch; record current through this circuit.

6. Calculate average current for two situations.

7. Calculate average charge moved in one second.

8. Calculate number of electrons flowing through circuit in one second. (1 coulomb = 6.24×10^{18} electrons.)

9. Answer question in Data Table.

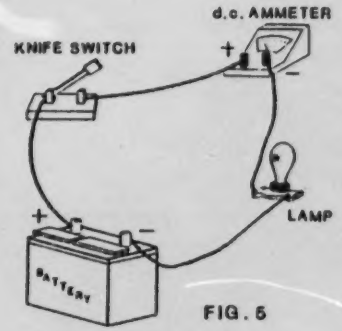


FIG. 5

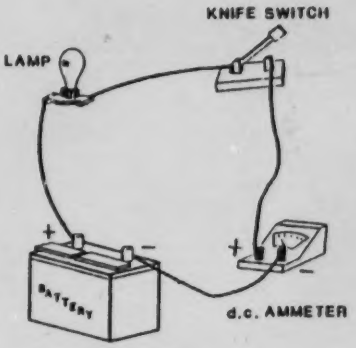
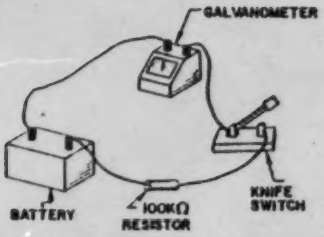


FIG. 6

DIRECTION OF CURRENT FLOW



MATERIALS

12-volt battery or d.c. power supply.
Clip leads.
Knife switch.
Galvanometer.
100-kΩ resistor.

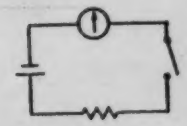


FIG. 7

PROCEDURE

1. Connect circuit as shown in Figure 6.
2. Close switch; note direction of deflection of galvanometer needle.
3. Indicate in Figure 7 direction of both conventional and electron current. Record direction and magnitude of needle deflection.
4. Reverse polarity of battery connections; repeat experiment.
5. Explain polarity and its importance.

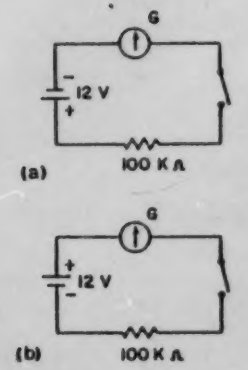


FIG. 8

DATA TABLE
Current Measurement Current from first procedure, step 3 _____. Current from first procedure, step 5 _____. Average current _____. What was shown by reversing circuit? _____ _____ _____ Average charge moved in one second _____. Number of electrons flowing in one second _____. Direction of Current Flow First Trial Indicate in Figure 7a direction of conventional and electron current flow. Direction of needle deflection _____. Magnitude of needle deflection _____. Second Trial Indicate in Figure 7b direction of conventional and electron current flow. Direction of needle deflection _____. Magnitude of needle deflection _____.

SELF CHECK

1. Define:

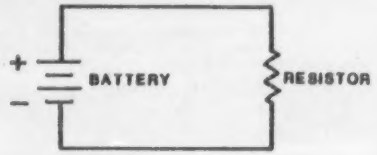
   Current.
   Direct current.
   Alternating current.
   Ampere.
   Electron flow.
   Conventional current flow.

2. A current of 16 amps flows through a conductor. How much charge is transferred in 25 ms?

3. If one minute is required for 10 C of charge to flow through a conductor, what is the current?

4. How much time is required for a current of 25 mA to transfer a charge of 0.5 C?

5. In the below circuit sketch, indicate (1) direction of electron flow with a solid line and (2) direction of conventional current flow with a dashed line.



6. Draw a graph showing alternating current as a function of time in an a.c. circuit.

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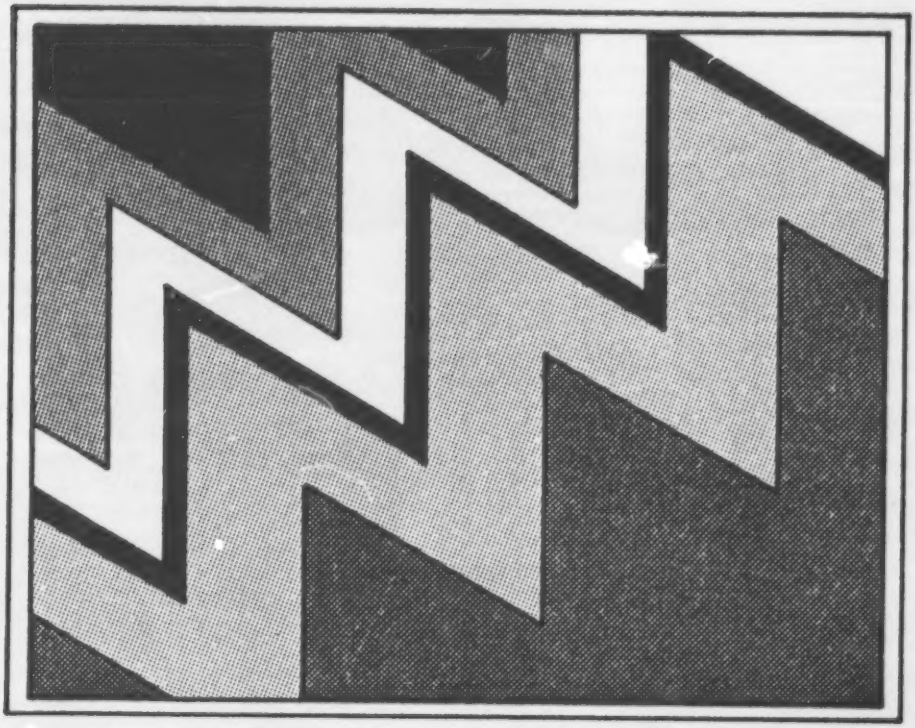
UNIFIED TECHNICAL CONCEPTS

MODULE 3 E 2

FREQUENCY



ELECTRICAL APPLICATION OF RATE

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TERC - SW

INTRODUCTION

The membrane of a speaker vibrates 10,000 times in 20 seconds to produce sound of a certain pitch. The frequency of vibration of the speaker (and the resultant sound wave), therefore, is 500 cycles per second. Frequency is a quantity that provides a measure of the number of repetitive events that occur in a given time and, therefore, is a rate at which certain events occur.

In this module, the student will learn how the quantities frequency and time relate to each other in repetitive events. The importance of known frequencies for time measurement will be noted. In addition, the concept of spatial frequency will be introduced. In the laboratory, the student will measure the characteristic frequencies associated with some simple electrical, mechanical, and optical systems.

PREREQUISITES

The student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following concepts:

  Temporal frequency.
  Spatial frequency.
  Period.

- Given one of these two quantities, find the other:

  Frequency.
  Period.

- Given one of these two quantities, find the other:

  Spatial frequency.
  Spatial separation.

- Find the smallest interval of time that can be measured with different types of timekeeping devices.
- Given the appropriate equipment, measure the period of a pendulum and of an electrical wave.
- Given the appropriate equipment, determine the spatial frequency of the grooves in a diffraction grating, and calculate their spatial separation.

DISCUSSION

Frequency is a measure of the repetitive nature of events or of objects. Temporal frequency describes events that repeat after sepcific intervals of time have elapsed. Events that can be described in terms of temporal frequencies inlcude water dripping from a faucet, the ticking of a clock, or the motion of a planet around a star.

Spatial frequency describes objects that repeat over specific distances. Stripes on a highway, keys on a piano, and bottles in a carton are objects that can be well described by spatial frequencies.

Some objects and events can be characterized by both spatial and temporal frequencies. A train moving through a crossing is an example of such a system. The cars of the train are spaced evenly and, therefore, repeat after a certain distance. Because the train is moving, however, cars also pass through the crossing at definite, equal time intervals.

TEMPORAL FREQUENCY

Temporal frequency is defined as the rate at which a repetitive event occurs, as indicated by Equation 1.

$$f = \frac{n}{t} \qquad (1)$$

where

f = temporal frequency.
n = number of repetitions.
t = time in seconds.

Example A illustrates how the equation for frequency is used to solve a problem. (Note: Hereinafter the term "frequency" will be used to refer to temporal frequencies.)

EXAMPLE A: FREQUENCY OF AN ELECTRICAL WAVE

Given: A commercial generator produces 240 electrical waves in four seconds and operates for one hour.

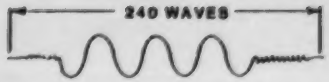
Find: (a) The frequency of the waves.
(b) The number of waves produced in this time.

Solution: (a) Use Equation 1.

$$f = \frac{n}{t}$$

$$f = \frac{240 \text{ waves}}{4 \text{ seconds}}$$

$$f = 60 \text{ waves/s}$$



(b)

$$f = \frac{n}{t}$$

$\therefore n = f\,t$ (multiplying both sides by time)

$$n = (60 \frac{\text{waves}}{\text{s}})(1 \text{ hour})$$

$$= (60 \frac{\text{waves}}{\text{s}})(1 \text{ hour})(\frac{60 \text{ min}}{1 \text{ hour}})(\frac{60 \text{ s}}{1 \text{ min}})$$

$$n = 216{,}000 \text{ waves}$$

Example A shows hot to calculate the frequency of a repetitive event and demonstrates how a knowledge of the frequency allows the calculation of the number of events occurring in a given time.

The standard units of frequency are the same in the mks, cgs, and English measuring systems. Frequency is measured in Hertz (Hz), where —

$$1 \text{ Hz} = 1 \text{ cycle/second} = 1 \text{ event/second.}$$

"Cycles" or "events" cannot be expressed in terms of units of mass, displacement, or time; therefore, in a strict sense, frequency is measured in reciprocal time units:

$$1 \text{ Hz} = 1/\text{s} = 1 \text{ s}^{-1}$$

These units immediately lead to Equation 2, another form for the definition of frequency, completely equivalent to Equation 1:

$$f = \frac{1}{T} \qquad (2)$$

where

T = period (time) for one event.

Equation 2 is solved for T as shown in Equation 3.

$$fT = 1 \quad \text{(Multiplying both sides by T)}$$

$$T = \frac{1}{f} \quad \text{(Dividing both sides by f)} \qquad (3)$$

Thus, the period of any repetitive event is simply the reciprocal of its frequency.

EXAMPLE 8: PERIOD OF VIBRATION OF A TUNING FORK

Given: A tuning fork vibrates at a frequency of 800 Hz.

Find: The period of vibration of the tuning fork.

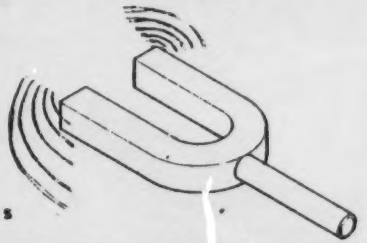
Solution: From Equation 3:

$$T = \frac{1}{f}$$

$$= \frac{1}{800\ \text{Hz}}$$

$$T = 0.00125\ \text{s}$$

Note: $\frac{1}{\text{Hz}} = \frac{1}{1/\text{s}} = \text{s}$



Many common energy systems exhibit repetitive motion and, hence, can be characterized by a specific period and frequency. Probably the most familiar such system is the simple pendulum, which consists of a massive object suspended by a light rod or string. If the pendulum is allowed to swing freely, its period will be constant. If the pendulum swings only through a small angle (approximately 5° or less), the period of the pendulum depends only upon its length and the acceleration of gravity as shown in Equation 4.

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad (4)$$

where

ℓ = length of pendulum.
g = acceleration of gravity.

For the past three centuries, the major application of the pendulum has been as a timer. Recently, however, pendula have been used to determine the acceleration of gravity to a high degree of precision. This determination is technologically important for oil exploration, since the acceleration of gravity

over a large oil deposit often is somewhat less than the nominal value of 980 cm/s^2.

EXAMPLE C: FREQUENCY OF A SIMPLE PENDULUM

Given: A simple pendulum of length 20 cm swings in a region where the acceleration of gravity is 965 cm/s^2.

Find: Frequency of the pendulum.

Solution: First, use Equation 4 to find the period:

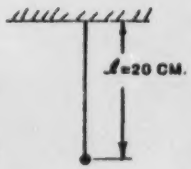
$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$= 2\pi\sqrt{\frac{20\ cm}{965\ cm/s^2}}$$

$$= 2\pi\ \ 0.0208\ s^2$$

$$= 2\pi\ (0.144\ s)$$

$$T = 0.904\ s$$



Use Equation 2 to find the frequency?

$$f = \frac{1}{T}$$

$$= \frac{1}{0.904\ s}$$

$$= 1.10/s$$

$$f = 1.10\ Hz$$

The frequency of the pendulum is 1.10 Hz.

By far, the most common (and oldest) application of the frequency concept is the measurement of time. In this application, time is measured by counting repetitive phenomena. If the frequency of the repetitive event is constant, then the period of the motion defines a basic unit of time. The smallest unit of time that can be measured by any repetitive event is equal to the period of the event; consequently, the higher the frequency of any repetitive phenomenon, the smaller the basic unit of time that it can measure — thus allowing more precise time measurements.

The first repetitive events used as time standards were the motions of the heavenly bodies. These motions were employed to define three basic units of time: the day (based upon the rotation of the earth on its axis), the year (revolution of the earth

about the sun), and the month (motion of the moon with respect to the sun and the earth).

These standards of time were impractical for two reasons. First, the extremely small frequencies of the repetitive motions involved caused the time units to be too long for many applications. Second, the very nature of the motions made very difficult the precise measurements of their periods.

Many clever devices were developed to subdivide the smallest of these units (the day) into even smaller divisions (hours, minutes, and seconds). These devices included the sun dial, the hourglass and the water clock; but the basic unit of time remained the day.

In the early 16th century, Galileo studied the motion of simple pendula and deduced Equation 4. This simple discovery revolutionized time measurement. The smallest unit of time that could be measured with a pendulum (i.e., the period of the pendulum) is on the order of one second. Pendulum clocks were extremely reliable and remained the major timekeeping instruments for several hundred years. Their major drawbacks were their fragility and lack of portability.

Both of these problems were overcome with the introduction of the next timekeeping standard. A small balance wheel was mounted on an axle and caused to rotate back and forth by a fine spring. With suitable values for the mass and radius of the wheel and for the tension of the spring, the balance wheel could be made to oscillate many times each second. In addition, the balance-wheel mechanism proved far more rugged than pendulum clocks. The balance wheel also required much less room than the pendulum, making small clocks and watches possible.

Many modern clocks use the frequency of alternating voltage to determine their basic time unit. The frequency of this eelctrical votage is 60 Hz (50 Hz in most foreign countries) and divides the second even more finely than the balance wheel.

The natural vibrations of a quartz crystal provide even finer division of the second. Typical natural frequencies for these crystals are from one to ten megahertz (1 MHz = 10^6 Hz), allowing basic time units of $10^{-6} - 10^{-5}$ seconds, respectively.

The ultimate in timekeeping precision is presently the atomic clock. This device uses the frequency of electromagnetic radiation emitted from certain atoms or molecules (typically cesium or ammonia) for a time standard. The frequencies involved are generally in the range from 1 – 10 gigahertz (1 GHz – 10^9 Hz). One such clock, operated by the National Bureau of Standards, is accurate to within a few seconds in one million years.

SPATIAL FREQUENCY

Any object or event which repeats after a given distance can be characterized by a spatial frequency. The definition of spatial frequency can be stated in a form similar to Equation 1, as indicated by Equation 5.

$$k = \frac{n}{d} \qquad (5)$$

where

k = spatial frequency.
n = number of spatial events that occur in distance d.
d = distance.

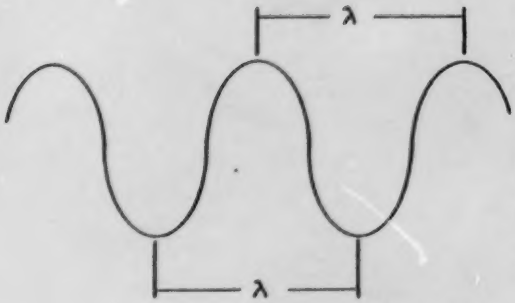


FIG. 1

The spatial frequence also can be expressed in terms of the spatial separation; that is, the distance between two consecutive objects or events. Spatial separation is defined by Equation 6.

$$k = \frac{1}{d} \qquad (6)$$

Notice the similarity between Equations 2 and 6. In Equation 6 the units of spatial frequency are reciprocal distance units; spatial frequency, therefore, has different units in each of the three systems of movement.

One class of phenomena that is readily described by spatial frequencies is wave motion. The spatial separation of waves (for instance, the sound waves in Figure 1) is the distance between two consecutive peaks (or two consecutive troughs) and is called the "wavelength." Example D shows how the spatial frequency of waves is calculated.

EXAMPLE D : SPATIAL FREQUENCY OF WATER WAVES	
Given:	Ocean waves have peaks every 10 m.
Find:	The spatial frequency of these waves.
Solution:	Wavelength = spatial separation = 10 m.
	Then —

$$k = \frac{1}{d}$$

$$k = \frac{1}{10\ m}$$

$$= 0.1/m$$

$$k = 0.1\ m^{-1}$$

The concept of spatial frequency often is applied to optical instruments. Photographic film consists of fine grains of light-sensitive material (usually silver nitrate) on glass or cellulose. When exposed to light, these grains darken in proportion to the amount of light incident upon them to form the familiar photographic negative. Naturally, the clarity of detail in the negative will be better if the film contains many small, closely-packed grains, rather than large grains not so closely packed. The spatial frequency of the grains on the film is then a measure of the ultimate clarity (or resolution) of the negative. The greater the spatial frequency, the better the film resolution will be.

Another device called a "diffraction grating" can be used to bend, or deviate, a beam of light. The grating is a glass or plastic slide with many long, parallel scratches or grooves on its face. If the grating is illuminated with a beam of light of a single wavelength, it will break the beam into a diffraction pattern of alternating light and dark bands (Figure 2).

The brightest band is in the center of the pattern and is called the "central band." The distance from the central band to the nth outlying band (x in Figure 2) is related to the wavelength of the light and the distance ℓ from screen to grating by Equation 7.

$$nk\lambda = \frac{x}{\ell} \qquad (7)$$

where

- k = spatial frequency of the grooves on the grating.
- λ = wavelength of light.
- x = distance from central band to outlying band.
- ℓ = distance from screen to grating.

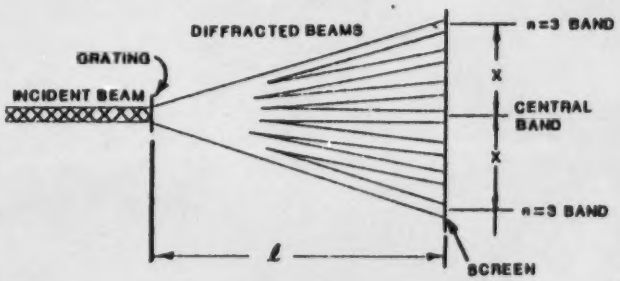


FIG. 2

The diffraction grating is treated in more detail in Module 13L12.

EXAMPLE E : SPATIAL FREQUENCY OF DIFFRACTION GRATING GROOVES

Given: In Figure 2, $\ell = 5$ m, $\lambda = 5 \times 10^{-7}$ m, and the distance from the central band to the third outlying bright band is 3 m (0.03 m).

Find: The spatial frequency of the grating's grooves.

Solution: Note that $n = 3$ (third band) and that $x = 0.03$ m. From Equation 7:

$$n k \lambda = \frac{x}{\ell}$$

$$k = \frac{x}{n \lambda \ell} \text{ (dividing both sides by } n\lambda)$$

$$= \frac{(0.03 \text{ m})}{3\ (5 \times 10^{-7} \text{ m})(5 \text{ m})}$$

$$k = 4000/\text{m}$$

The grating has 4000 grooves per meter.

SUMMARY

Frequency is the rate at which events occur. "Temporal" frequency refers to events that repeat after specific time intervals have elapsed. Examples of temporal frequency are timekeeping devices, electrical waves, vibrations of a tuning fork, and repetition of a pendulum's motion. "Spatial" frequency describes objects that repeat over specific distances; for example, wavelength and the groove frequency of a diffraction grating.

FREQUENCY OF ELECTRICAL OSCILLATIONS

MATERIALS

Signal generator.
Oscilloscope.
2 wire leads.

PROCEDURE

1. After instructor has adjusted oscilloscope in such a manner that electrical wave can be seen on the screen, set signal generator frequency to 200 Hz.

2. Use oscilloscope time base (s/div switch) to adjust the size of the wave until one wavelength is spread evenly over the entire screen. Record s/div switch setting in Data Table 1. (Note: "m/s" means milliseconds — 1 ms = 10^{-3} s.)

3. Measure number of divisions in one wavelength λ of electrical wave. Record this value in Data Table 1.

4. Calculate the period T and frequency f of the electrical wave.

5. Repeat this procedure for signal generator frequencies of 400 Hz, 500 Hz, and 1000 Hz.

DATA TABLE 1

	Signal Generator Frequency	200 Hz	400 Hz	500 Hz	1000 Hz
	s/div	_____ s	_____ s	_____ s	_____ s
	λ	_____	_____	_____	_____
	T = s/div x λ	_____ s	_____ s	_____ s	_____ s
	f = 1/T	_____ Hz	_____ Hz	_____ Hz	_____ Hz

PERIOD OF A PENDULUM

MATERIALS

Pendulum (200 g hooked mass tied to approximately 150 cm of light string).
Stopwatch or electric timer.
Meter stick.

PROCEDURE

1. Suspend pendulum in a doorway. Measure and record length ℓ (distance from center of mass to suspension point at top of doorway).

2. Displace mass approximately 10 cm horizontally. Simultaneously release mass and start the timer. Stop timer after pendulum has completed 20 swings. (Mass must return to original release point to complete a swing.) Record elapsed time t in Data Table 2.

3. Calculate period T from t and number of swings.

4. Calcualte period T from ℓ, and acceleration of gravity ($g = 980\ cm/s^2$), using Equation 4.

DATA TABLE 2
Length of pendulum = ℓ = _____ cm
Number of swings = n = 20
Elapsed time = t = _____ s
$T = \frac{t}{20}$ = _____ s
$T = 2\pi\sqrt{\frac{\ell}{g}}$ = _____ s

SPATIAL FREQUENCY OF A DIFFRACTION GRATING

MATERIALS

He-Ne laser.
Student diffraction grating.

Metric ruler or tape measure.
White cardboard screen.

PROCEDURE

1. Set up laser, diffraction grating, and screen as shown in Figure 3. Distance should be at least 2 m. Turn on laser and align the beam so that a diffraction pattern appears on the screen.

   CAUTION: LASERS PRODUCE A HIGH INTENSITY BEAM OF LIGHT WHICH CAN DAMAGE YOUR RETINA IF DIRECTED INTO YOUR EYE. DO NOT LOOK DIRECTLY INTO THE LASER BEAM.

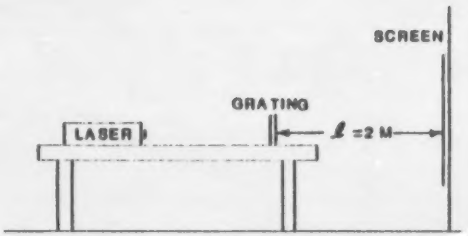


FIG. 3

2. Measure, and record distance ℓ in Data Table 3.

3. Measure and record the distance x from the center of the central band to the center of the fourth band to its left or right. (The value of n in Equation 7 for this band is then equal to 4.)

4. The He-Ne laser produces light of a single wavelengh, $\lambda = 6.328 \times 10^{-5}$ cm. Using this value for λ and the values of n, k, and ℓ already determined, calculate the spatial frequency k of the grooves on the grating.

5. Using this value of k, calculate the spatial separation of consecutive grooves d.

DATA TABLE 3	
$\ell =$ _____ cm	$d = \frac{1}{k} =$ _____ cm
$\lambda = 6.328 \times 10^{-5}$ cm	$x =$ _____ cm
$k = \frac{x}{n\lambda\ell} =$ _____/cm	$n = 4$

SELF CHECK

1. Define the following:

   Temporal frequency.
   Period.
   Spatial frequency.

2. A train of identical boxcars travels at constant speed past an observer. The observer sees a car go by every four seconds. With what temporal frequency do the cars pass the observer? If each car is 20 m long, what is the spatial frequency of the boxcars?

3. A diffraction grating has a groove spacing of 0.002 mm. What is its spatial frequency?

4. Find the period of waves with the following frequencies:

   a. 50 Hz.
   b. 50 KHz.
   c. 50 MHz.

5. What is the smallest time measurable with each of the following devices:

   A pendulum of length 1.5 m (assume $g = 9.8\ m/s^2$).
   A 5.2 MHz quartz crystal.
   An atomic clock that uses electromagnetic radiation of frequency 5×10^{14} Hz as its reference.

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See especially Sections 51 and 67.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 T 1

HEATING, COOLING RATE

THERMAL APPLICATION OF RATE

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Heat energy can be transferred into or out of an area so as to increase or decrease its temperature. Such heating or cooling is familiar. Consider heating a pot of food on a stove and then cooling the leftovers in the refrigerator. The heating or cooling rate is the change in temperature divided by the elapsed time. It is measured in units of degrees temperature per unit time.

In this module, heating and cooling rates under various conditions will be measured.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

To complete this module, the student should be able to:

- Define heating and cooling rate by means of an equation, stating the appropriate units, and how to distinguish between a heating rate and a cooling rate.
- Define three processes by which heat energy is transferred to produce heating or cooling.
- Calculate and plot the cooling rate versus time of several common systems given the required materials, temperatures and conditions.
- Calculate the final temperature of an object after cooling, given materials, temperatures and time span.
- Given proper equipment, measure heating and cooling rates of various materials.

DISCUSSION

In this module, the rate at which the temperature of a material changes, called the heating or cooling rate, will be discussed. The rate is defined by the equation:

$$\text{Rate of heating or cooling} = \frac{\text{temperature change}}{\text{elapsed time}} \qquad (1)$$

In symbols this becomes:

$$R = \frac{T_2 - T_1}{t_2 - t_1} \qquad (2)$$

where

R = rate.
T_1 = temperature at time t_1.
T_2 = temperature at time t_2.

Assume that t_1 is the initial starting time. Then $t_2 - t_1$ is the elapsed time and will always be a positive number. If the temperature is increasing, T_2 is greater than T_1. Then Equation 2 defines a heating rate and the sign of the quotient will be positive. If T_2 is less than T_1, the sign in Equation 2 will be negative. Then Equation 2 defines a cooling rate.

The units of heating or cooling rate are units of temperature per unit time, for example °C per second or °F per hour.

Practical examples of heating and cooling are not difficult to find. Consider how the temperature in a room increases when the furnace turns on (Example A) or how the temperature of a pot of water increases when one turns on the stove beneath it. Simi-

EXAMPLE A: HEATING RATE OF A HOUSE

Given: On a chilly winter morning, the furnace has gone out and the temperature is 40° F. The furnace is started for 30 minutes.

Find: Heating rate if final temperature of the house is 70° F.

Solution: Using Equation 1, the elapsed time $t_2 - t_1$ is 30 minutes. The change in temperature $T_2 - T_1$ is 70° F - 40° F = 30° F. Then the heating rate is 30° F/30 minutes or 1° F per minute.

larly examples of cooling are common. Food placed in the refrigerator will cool from room temperature to the temperature of the refrigerator.

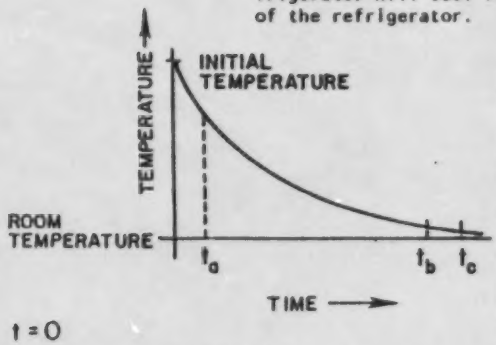

Fig. 1

In most cases the rate of heating or cooling will not be constant. For example, if a pan of hot water sits in a room to cool, the rate of cooling will be highest at first, when the water is still well above room temperature. When the water approaches room temperature, the cooling rate becomes slower. This is sketched schematically in Figure 1, which shows the cooling rate at various times. The rate of cooling is highest at $t = 0$. As the temperature comes closer to room temperature, the cooling rate decreases. In the elapsed time from $t = 0$ to $t = t_a$, the temperature drops considerably. In an equal elapsed time interval from t_b to t_c, when the sample temperature has come closer to room temperature, the change in temperature is less, and therefore, the cooling rate is lower (Example B).

EXAMPLE B: COOLING OF THE EARTH'S SURFACE

Given: Data included in the narrative solution.

Find: The cooling of the earth's surface after sunset. The decrease in temperature after sunset on a clear, windless night is due to removal of surface heat by radiation. This leads to a decrease in temperature which is proportional to the square root of the elapsed time after sunset. (This result is simply stated here without proof.) The temperature, T, is given by the equation:

$$T = T_s - K\sqrt{t} \qquad (3)$$

Solution: In this equation T_s is the temperature at sunset, K is a factor of proportionality which describes the cooling rate and t is the elapsed time after sunset. Consider a night on which the temperature at sunset is 70° F and the temperature at dawn, assumed to be 12 hours later, is 50° F. Equation 3 yields a curve of the form shown in Figure 2. This shows the temperature as a function of time during the night. The initial rate of decrease of temperature is fairly fast but becomes slower. The lowest temperature is reached just before dawn.

This, of course, is an approximation, and factors such as winds, clouds, motion of air masses and so on can change the shape of the curve from the idealized curve shown in Figure 2.

If a substance undergoes a change of phase at a certain temperature, that is, if it melts or freezes, boils or condenses, the form of the heating/cooling curve giving temperature as a function of time will be different. This is illustrated in Figure 3, and would apply to a sample of ice initially at a temperature below 0° C, subjected to a constant heating rate starting at time t = 0. From time 0 to t_a (section A of the graph in Figure 3) the ice is warming at a constant rate until, at time t_a, it reaches 0° C. At that point, the temperature remains constant while heat energy is used to change the phase of the substance. The heat energy delivered to the ice is used to melt the ice. The temperature does not rise until all the ice is melted (section B of the curve in Figure 3). At time t_b, after all the ice is melted, the temperature will begin to rise again (section C of the curve). The heating rate is different than that in section A because the specific heat of liquid water is different from the specific heat of ice. This type of behavior is a common phenomenon observed whenever there is melting, freezing, boiling or condensation of a vapor and a break in the cooling curve and the temperature will remain constant until the change of phase is completed. Thus, the mixture of water and ice remains at the constant temperautre 0° C until all the ice is melted.

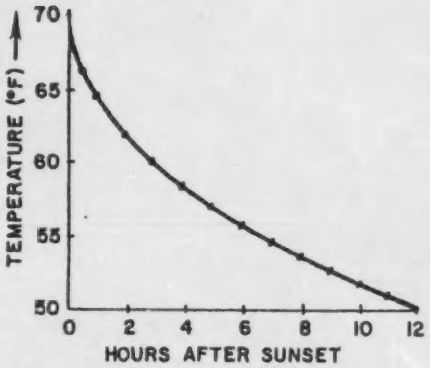

Fig. 2

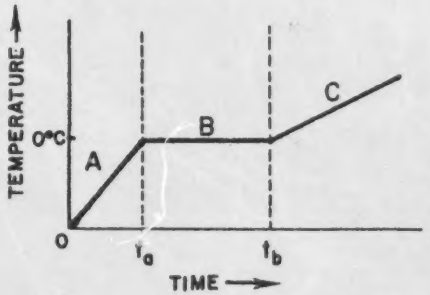

Fig. 3

Heat can be transferred from one place to another by three main processes:

Conduction.
Convection.
Radiation.

The first of these, conduction, involves transfer of heat energy through a material. The material can be solid, liquid or gaseous, and the heat is transferred through the material by atomic motions.

Convection occurs when heat is transferred through physical motion of a body of fluid, either a liquid or a gas. An example of convection by a liquid is provided by the operation of a hot water house furnace. Water is heated in the boiler and is transferred through pipes to a radiator where it gives up its heat.

returning to the furnace to be reheated. This is an example of heat transmission by physical motion of the heated fluid, that is by convection.

The third way in which heat may be transmitted from one place to another involves no material. This method is called radiation. Heat transfer by radiation is familiar, as the energy in sunlight is transferred by radiation, and the heat delivered by the sun can be felt. Surfaces of all bodies emit energy by radiation and the amount of energy radiated from a surface increases sharply as the temperature of the surface increases. An example would be heat energy radiated from a fireplace.

MATERIALS

Bunsen burner.
Beaker.
Thermometer 10° C to 110° C.
Stopwatch.
Double-walled beaker.
Antifreeze (ethylene glycol base).
Stand to support beaker over burner.
Graph paper.

PROCEDURE

1. Heat a beaker of water on burner until it reaches boiling temperature. Remove flame and record temperature and start the stopwatch. Keep the stopwatch running as the temperature is recorded every 20 seconds.
2. Measure temperature at recorded time intervals until the temperature of water is within ten degrees of room temperature. Plot temperature as a function of time on a piece of graph paper. Calculate the cooling rate for each pair of adjacent time intervals. Plot the cooling rate as a function of time on a piece of graph paper.
3. Next prepare a mixture of 50% antifreeze and 50% water, place this in the outer section of a double-walled beaker. Fill the inner section with plain water. Heat the entire apparatus over the burner. The experimental arrangement is sketched in Figure 4.
4. Gently stir with the thermometer continuously, and measure the temperature of the antifreeze-water mixture. Observe the temperature as a function of time, using the stopwatch, as the water in the inner beaker heats to its boiling point and then begins to boil away. Take temperature readings at appropriate time intervals.
5. Continue taking temperature readings while the water in the inner beaker is boiling. The temperature in the outer portion of the beaker should begin to rise again after all the water is gone. Stop heating the beaker before the temperature in the outer beaker reaches 227° F, the temperature at which a 50% mixture of antifreeze and water will begin to boil.
6. Plot the temperature of antifreeze-water mixture as a function of time on a piece of graph paper. From pairs of ad-

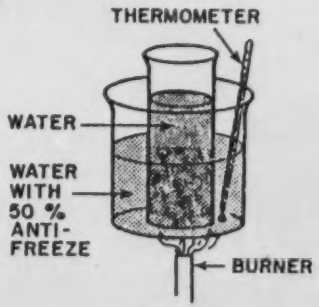

Fig. 4

jacent temperature measurements at different times, calculate the heating rate, using Equation 2. The elapsed time will be time between successive measurements of temperature. Then plot the heating rate versus time.

7. Interpret the form of the curve obtained. Discuss why the heating rate dropped to a lower value while the water in the internal part of the beaker was boiling.

SELF CHECK

1. State the mathematical formula for the heating or cooling rate. What are the appropriate units? How does one determine if the rate is to be interpreted as heating or cooling?
2. State three methods by which heat energy is transferred to produce heating or cooling.
3. Suppose that one places a bowl of jello in the refrigerator to cool. The initial temperature of the jello is 200° F. After 5 minutes its temperature is 150° F; after 10 minutes 110° F; after 15 minutes 80° F; after 20 minutes 60° F; after 25 minutes 50° F; after 30 minutes 44° F; after 35 minutes 40° F; and after 40 minutes 38° F. Plot the cooling rate versus time.
4. Suppose that one places a steel ingot in a high temperature furnace. In the early stages of the heating, the heating rate is nearly constant. If after the first 10 seconds the temperature of the ingot goes from 70° F (room temperature) to 200° F, what is the temperature of the ingot after 20 seconds?

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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 3 T 2

RATE OF HEAT CONDUCTION

THERMAL APPLICATION OF RATE

Technical Education Research Center – Southwest

TERC – SW

INTRODUCTION

Heat is conducted through matter by the increased motion of atoms and molecules in the material. Heat conduction occurs whenever a difference in temperature exists, and the direction of heat flow is always in the direction of decreasing temperature. Temperature difference is the force-like quantity which produces motion in the thermal system.

This module treats the rate of heat conduction through solid materials, taking into account the physical quantities affecting rate, and relating them through a coefficient of thermal conductivity or an average coefficient of heat transmission. The use of insulation in reducing the rate of heat conduction is emphasized. In the laboratory, the student will determine the rate of heat conduction for several materials.

PREREQUISITES

Student should have completed Concept Module 3-0, "Rate."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given an explanation of heat conduction, including comments about kinetic energy transfer, importance of temperature difference and conditions that determine direction of heat conduction.
- State the equation that defines the rate of heat conduction, and describe briefly each factor.
- Given appropriate information, determine which materials have a higher value of heat conduction.
- Given the appropriate equipment, determine the rate of heat conduction for various materials.

DISCUSSION

Heat energy can be transferred from one point to another by the process of thermal conduction, also called heat conduction. Heat conduction can be demonstrated by a simple experiment in which one end of a metal rod is placed in a flame. That end of the rod will be heated by contact with the flame. Heat will be transferred down the length of the rod by the process of heat conduction, so that eventually the entire rod will become hotter. The end of the rod in the flame always will be the hotter part of the rod. The end farthest from the flame will always remain the cooler part. But all parts of the rod will be hotter than the original unheated rod.

The process by which heat is conducted down the rod is atomic. The atoms at the hot end of the rod vibrate more rapidly as the temperature increases. The more rapidly-vibrating atoms collide with their more slowly moving neighbors and transfer some of their vibrational energy. Those atomic neighbors, in turn, pass the energy along to those farther removed from the heat source. Hence, the energy of thermal motion (the motion of atomic vibrations) is conducted from one atom to the next. In the conduction process, each individual atom remains near its original position, vibrating more vigorously as the temperature increases. Although the atoms remain relatively stationary, their increased vibrational energy is transferred; and thermal energy is conducted down the entire length of the rod, from hot end to cold end.

There are two other processes - convection and radiation - by which heat can be transferred from one place to another. In convection there is a net physical motion of the heated fluid (large groups of atoms or moleucles moving from one position to another); whereas, in conduction there is no net motion. Radiation transfers heat through empty space; that is, radiation does not require the presence of matter as do convection and conduction. For example, the earth receives radiated heat energy from the sun, and such energy had to pass through millions of miles of empty space (vacuum) before reaching earth.

Although heat conduction exists in solids, liquids or gases, in this module, solids as conductors of heat, will be addressed. Usually, liquids and gases are relatively poor conductors of heat, compared to many solids, particularly metallic solids. Also, discussion of conduction in gases or liquids is usually complicated by convection processes.

The conduction of heat may be expressed as a rate of energy transfer. The rate of heat conduction is given by Equation 1.

$$Q_H = \frac{H}{t} \qquad (1)$$

where

Q_H = rate of heat conduction.

H = quantity of heat energy conducted.

t = time elapsed.

The rate at which heat is conducted in a specific situation is determined by measuring the total amount of heat energy transferred in a given elapsed time. If heat energy is moving at a constant rate, Equation 1 gives a precise result. If heat energy is moving at different rates during the elapsed time, Equation 1 gives the average rate.

The units for the rate of heat conduction are energy divided by time. Thus, possible units are joules per second (one joule per second is identical to a watt), calories per second, or Btus per hour. Table 1 shows examples of the relations between some of these different units.

TABLE 1: UNITS FOR RATE OF HEAT CONDUCTION

Joule per second (watt)
Calorie per second
Btu per second
Btu per hour

Relation Between Units:

1 joule per second = 0.239 calorie per second = 0.000948 Btu per second = 3.41 Btu per hour.
1 calorie per second = 4.18 joule per second = 0.00396 Btu per second = 14.27 Btu per hour.
1 Btu per second = 1055 joule per second = 252 calorie per second = 3600 Btu per hour.
1 Btu per hour = 0.293 joule per second = 0.0701 calorie per second = 0.000278 Btu per second.

As indicated in Equation 1, heat conduction rate is the measured quantity of heat transferred divided by the time elapsed during transfer. Heat rate calculation is not always a simple task, because there may be several processes involved in the heat transfer. In particular, as already mentioned, heat can be transferred by convection, conduction and radiation. When heat conduction of a metallic rod is measured, the necessity may arise to separate out heat transfer by either convection or radiation (or both), either of which can affect the true measurements of heat conduction in the rod. Heat can be transferred away from the hot end of the rod by processes other than conduction, in such a manner that an over-

estimate of the amount of heat transferred down the rod by conduction could be made. In many practical cases, the heat input at one end of a sample and the heat conducted along the length of the sample can be accurate. In such cases, Equation 1 can be used to determine heat conduction rates. Example A demonstrates the use of Equation 1 in solving a problem.

EXAMPLE A: RATE OF HEAT TRANSFER INTO A CALORIMETER FROM WARM AIR SURROUNDING THE CALORIMETER

Given: In a calorimeter containing 100 g of water, the temperature of the water rises from 5°C to 11°C in 30 minutes. The water equivalent of the calorimeter is 50 calories per degree C.

Find: Rate of heat transfer into the calorimeter.

Solution: The heat gained by the water is as follows:

$$100\ g \times 1\ cal/g\ °C \times (11-5)\ °C = 600\ cal$$

The heat gained by the calorimeter is as follows:

$$50\ cal/°C \times (11-5)\ °C = 300\ cal$$

Thus, the total heat gained is 900 cal. The elapsed time is 30 minutes = 1800 seconds.

Equation:
$$Q_H = \frac{H}{t}$$
$$= \frac{900}{1800}$$
$$Q_H = 0.5\ cal/s$$

Thus, 0.5 cal/second flow from the warmer room (room air around the calorimeter) into the cooler calorimeter.

The flow of heat down a rod having one end heated in a flame is sketched in Figure 1. The end in the flame is heated to a temperature T_1. The end away from the flame is at a lower temperature T_2. The direction of conductive heat flow is shown by the arrow. This simple sketch shows two important facts about heat conduction:

There must be a temperature difference to produce heat flow by conduction.
Heat flows in the direction of decreasing temperature.

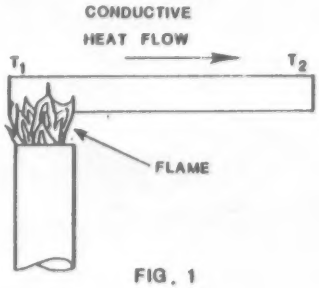


FIG. 1

The rate of heat conduction to the difference in temperature is related. Force-like quantities for heat conduction are the temperature differences. In a situation in which a solid material is being cooled by conduction of heat out of the material, the cooling rate is given by Equation 2 as follows:

$$Q_C = \frac{T_1 - T_2}{t_2 - t_1} \qquad (2)$$

where

Q_C = cooling rate.
T_2 = final temperature.
T_1 = initial temperature.
$t_2 - t_1$ = elapsed time.

For this cooling process, the rate of heat conduction is given by Equation 3.

$$Q_H = \frac{\Delta Q}{\Delta t} = \rho c V Q_C = \rho c V \frac{T_1 - T_2}{t_2 - t_1} \qquad (3)$$

where

ΔQ = temperature difference.
Δt = time difference.
Q_C = cooling rate.
ρ = density of material.
c = specific heat of material.
V = volume of material.

Note that ρV is equal to the mass of the material. Equation 3 shows that the rate of heat conduction, Q_H (for example, in calories per second), can be determined if the cooling rate, Q_C (for example, in °C/s), is known. Example B shows how Equation 3 is used to solve a problem.

EXAMPLE B: SOUP SPOONS ASSIST SOUP COOLING

Given: A bowl containing 300 cm^3 of soup that is too hot, at 80°C. The soup has properties similar to water (density = 1 g/cm^3, specific heat = 1 cal/g°C). A silver spoon is inserted into the bowl to conduct heat out at a rate of 40 cal/s.

Find: Temperature after 1 minute. (Neglect other heat losses such as conduction through the bowl or through the air.)

Solution: $Q_H = 40$ cal/s

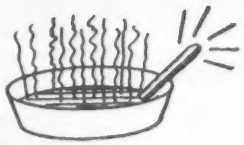
$$Q_H = \rho c V \frac{T_1 - T_2}{t_2 \quad t_1} \quad \text{(Equation 3)}$$

$$40 = \frac{1 \times 1 \times 300\ (80 - T_2)}{60s}$$

$$40 = 400 - 5\ T_2$$

or

$T_2 = 72°C$, final temperature.



HEAT CONDUCTION IN SPECIFIC MATERIALS

A slab of material (Figure 2) with a thickness, d, and a cross-sectional area, A, conducts heat from one side to the other if the temperatures of the two sides are different. If the temperature, T_1, of the left surface is higher than the temperature, T_2, of the right surface, heat is conducted in a direction from left to right, as shown by the arrows. Temperature difference is the driving force for thermal conduction. The total rate of heat flow from one side to the other depends upon a number of factors: the nature of the material, the surface area, A, the thickness, d, and the temperature difference. The rate of heat flow, Q_H, is given by Equation 4.

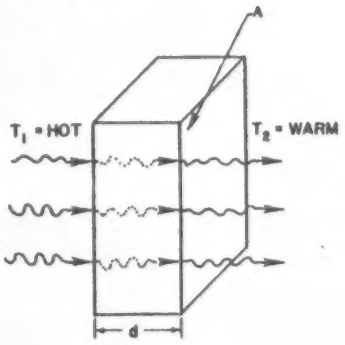


FIG. 2

$$Q_H = \frac{kA\ (T_1 - T_2)}{d} \qquad (4)$$

where

k = coefficient of thermal conductivity for the material.
A = cross-sectional area of material.
d = distance, or thickness of material.

Constant, k, is called the "thermal conductivity" of the material, and has units of heat flow rate per unit length, per unit temperature difference; that is, in calories per second per meter per degree, or equivalent units in other systems of measurement. Metallic materials are good thermal conductors; that is, they have high values of thermal conductivity, because electrons in a metal are free to move, and can travel through the metal, carrying heat energy as increased kinetic energy. This form of energy is in addition to the transfer of heat by the vibrations of atoms, which do not move through the solid. In insulating ma-

terials, such as glass or rubber, electrons are not free to move; and there is not the additional contribution of heat energy carried by electrons. If in addition then, the atoms in such substances as glass or rubber are tightly bound and cannot vibrate easily, the thermal conductivity generally is much lower than that of metals. As a consequence, insulators are not good heat conductors. Among metals, good heat conducting materials such as silver and copper have very high values of thermal conductivity, and metals such as steal have lower values.

Equation 4 assumes that the two sides of the slab of material have constant temperatures. A resonably good approximation to constant temperatures would be the case of an ideal house wall (no windows or doors) in winter. The inside of the wall is maintained at a constant temperature by the furnace in the house. The outside of the wall is maintained at the temperature of the outside air, which generally changes slowly. Thus, Equation 4 has direct application to problems such as this one, in which shapes are geometrically simple; temperature is constant over large areas; and material composition is homogeneous throughout the slab.

HEAT CONDUCTION IN SPECIFIC MATERIALS

The situation becomes more complicated with structures of practical engineering interest, such as real walls with windows and doors. The windows and doors may be surrounded by small cracks, through which heat can be lost by flow of air (convection). Walls may consist of several layers of various types of material. For such engineering materials, an average coefficient of heat transmission, denoted U, is commonly defined. The rate of heat conduction is given by Equation 5.

$$Q_H = UA\ (T_1 - T_2) \qquad (5)$$

where

U = average coefficient of heat transmission.
A = area.
$T_1 - T_2$ = temperature difference between two sides of structure.

If U were applied to a simple homogenious material as represented in Equation 4, U would be equal to the thermal conductivity divided by the thickness. Thus, the coefficient of heat transmission, U, generally is more applicable (if it is known) because it describes heat flow through complicated structures, such as walls consisting of layers of plasterboard, insulation and brick.

The units for U are commonly Btu/h x ft^2 x °F. Although this unit is not expressed in the SI system, it is the unit most often used by engineers and designers who work with building insulation.

Table 2 presents some values of U for various types of construction elements. The values are typical for homes of woodframe construction, and are given for various amounts of insulation in the ceiling and walls. These values should be considered as representative. Exact values will depend on the details of material, construction, etc. Example C shows the use of Table 2 in solving a problem.

TABLE 2: COEFFICIENT OF HEAT TRANSMISSION		
Walls	Insulation thickness	U_{wall}*
	0 inch	0.240
	1 inch	0.130
	2 inches	0.097
	3 inches	0.072
Ceilings	Insulation thickness	$U_{ceiling}$*
	0 inch	0.60
	2 inches	0.12
	4 inches	0.061
	6 inches	0.051
Windows	Window type	U_{window}*
	Single glass	1.13
	Thermopane	0.65
	Storm window	0.56
*U is in Btu/h x ft^2 x °F.		

EXAMPLE C: EFFECT OF INSULATION IN REDUCING CONDUCTIVE HEAT LOSS AND IN DECREASING FUEL USAGE IN HOUSES

Given: An older, wood-frame home, single story, of area 30 x 50 feet, with walls 8 feet high.

Find: Heat losses and effectiveness of insulation.

Solution: Many older homes have little or no ceiling insulation. Described in this example are the various terms leading to heat loss from a home and the effectiveness of adding ceiling insulation to reduce the heat loss. The heat loss, H, is given by:

$$H = (U_{wall} A_{wall} + U_{window} A_{window} + U_{ceiling} A_{ceiling} + \text{convection loss})(T_1 - T_2)$$

where

T_1 = indoor temperature.
T_2 = outdoor temperature.

The ceiling area is 1500 ft^2; the area of the walls, including windows, is as follows:

$$(2 \times 8 \times 30) + (2 \times 8 \times 50) = 480 + 800 = 1280 \text{ ft}^2$$

Typically, windows occupy about 20% of the wall area, in this case about 256 ft^2. Then the wall area is 1280 - 256 = 1024 ft^2.

A typical value for the convective loss term is about 213 Btu/h x °F, based upon one volume change of air per hour.

$$H = (1024\ U_{wall} + 256\ U_{window} + 1500\ U_{ceiling} + 213)(T_1 - T_2)$$

Assume that the house is reasonably well insulated except for the ceiling; that the walls have 2 inches of insulation; and that the house has storm windows, but no ceiling insulation. The values for U are obtained from Table 2 as follows:

$$H = (1024 \times 0.097 + 256 \times 0.56 + 1500 \times 0.60 + 213)(T_1 - T_2)$$
$$= (99.3 + 143.4 + 900 + 213)(T_1 - T_2)$$
$$= 1355.7\ (T_1 - T_2) \text{ Btu/h}$$

What would be the effect of adding 4 inches of ceiling insulation? The above expression would become:

$$H = (1024 \times 0.097 + 256 \times 0.56 + 1500 \times 0.061 + 213)(T_1 - T_2)$$
$$= (99.3 + 143.4 + 91.5 + 213)(T_1 - T_2)$$
$$H = 547.2\ (T_1 - T_2) \text{ Btu/h}$$

Thus, the simple addition of 4 inches of ceiling insulation would reduce the heat loss by about 60% in this example.

The exact values for heat conduction from a home will depend upon the details of the construction. For example, a two-story home will have a smaller ratio of ceiling area to wall area, so that the effect of adding ceiling insulation would be less. Still, the above example illustrates how insulation can reduce heat loss from buildings.

SUMMARY

Heat conduction (transfer) always proceeds in the direction of lower temperature. Conduction exists only when a temperature difference is present. The rate of heat conduction refers to the amount of heat transfer during a given time.

DETERMINING RATE OF HEAT CONDUCTION

MATERIALS

Copper bar (if not available, a copper tube will work).
Water and ice (about 50 g of ice in 250 cm^3 of water).
Two beakers or large styrofoam cups.
Balance.
Stopwatch.
Ringstand and clamps.
Stirrer.
Heating tape (about 100 - 200 watts).
Thermometer.

PROCEDURE

This experiment measures the quantity of heat conducted by a copper bar or tube in a measured time interval. Quantity of heat is determined by measuring the amount of ice melted in a mixture of water and ice at 0°C, with an experimental arrangement as shown in Figure 3. Note: A great deal of time can be saved during this experiment if steps 7-10 are performed during the same time that steps 1-6 are done. If this is done, two almost identical beakers of ice water must be used.

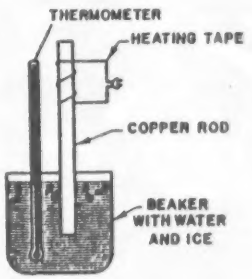


FIG. 3

1. Set up experimental arrangement shown in Figure 3, with copper bar supported by ringstand and clamps. Copper rod should have one end wrapped in heater tape and other end immersed in mixture of water and ice in beaker or styrofoam cups.

2. Stir water-ice mixture well, and use a thermometer to determine when mixture reaches 0°C.

3. Dip out ice in beaker, shake dry, and weigh on balance.

4. Immediately return measured weight of ice back to beaker, and, at the same time, turn on heating tape and start stopwatch.

5. Keep mixture well stirred, and observe amount of ice present.

When the last of the ice melts, stop stopwatch and record elapsed time in Data Table.

6. Measure heat flow into ice-water mixture in beaker from surrounding air. Prepare a second mixture of water and ice, and stir well to assure that its temperature is 0°C. (Remove copper bar for the remainder of this experiment.)

7. Dip out ice, shake dry, and weigh on balance. (This procedure is more accurate if weight of ice is close to that in step 3.)

8. As soon as ice is returned to water, start stopwatch. Keep mixture well stirred. Wait for a time interval equal to that observed in step 5.

9. Immediately at the end of time interval, dip out ice, shake dry, and weigh on balance.

10. Fill in Data Table, and calculate rate of heat conduction down copper bar. Value of heat flow rate in watts obtained in step K may be less than the number of watts delivered by heating tape. Why?

11. If a well-insulated calorimeter can be used in place of the beaker, perform only steps 1-5. Steps 6-10 are not necessary.

DATA TABLE	
A.	Mass of ice in step 3 _____ g.
B.	Time for ice to melt in step 5 _____ s.
C.	Heat energy needed to melt ice _____ cal. (Mass in A above x 80 cal/g.)
D.	Heat flow rate into ice-water mixture. (C ÷ B) _____ cal/s.
E.	Mass of ice in step 7 _____ g.
F.	Mass of ice in step 9 _____ g.
G.	Ice melted by heat flow from surroundings (E - F) _____ g.
H.	Energy to melt ice in G _____ cal (G x 80 cal/g).
I.	Heat flow rate from surroundings (H ÷ B) _____ cal/s.
J.	Heat flow rate by conduction through copper rod (D - I) _____ cal/s.
K.	Heat flow rate in watts (4.18 x J) _____ watts.

SELF CHECK

1. Explain the process of heat conduction. Include a model based upon the transfer of kinetic energy between neighboring atoms, and explain why heat conduction depends upon Δt and direction of heat flow.

2. State the mathematical relation that relates the rate of heat conduction in solids to the mass of material, specific heat, temperature difference, and time elapsed.

3. From each of the following pairs, indicate which member of the pair will have a higher value of heat conduction, when all other conditions are identical.

   Rubber - steel.
   Aluminum - glass.
   Copper - steel.
   An insulated section of a wall - an uninsulated section of a wall.

4. Suppose a pot of hot water at 100°C losses heat by conduction through the bottom of the pot. If there are 900 cm^3 of water in the pot, and if the temperature of the water after an elapsed time of 15 minutes is 60°C, what is the rate of heat conduction? (Remember, for water the density is 1 g/cm^3 and the specific heat is 1 calorie/g °C. Do not include any contribution from the specific heat of the pot itself.)

5. If a wall of a wood-frame house has no insulation and no windows and has an area of 480 ft^2, what is the rate of heat conduction through the wall when the inside temperature is 70°F and the outside temperature is 0°F? How much would this rate be reduced if the wall had 3 inches of insulation? (You may use Table 2.)

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